

WCES-2010

Primary teacher education students' ability to use functions as modeling tools

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Received November 4, 2009; revised December 7, 2009; accepted January 19, 2010

Abstract

The purpose of this study is to determine primary teacher education students' ability to use functions to solve problems containing a modelling dimension. This study was conducted with 51 first-year students at the Anadolu University. Data was gathered by a questionnaire and analysed according to some categories of techniques. The results indicate that students have many difficulties to perceive functions as modelling tools. Some of these difficulties appear to be related to the way the concept is introduced at the high school.

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Keywords: Mathematics; functions; mathematical modelling; daily life problem solving; primary teacher education.

1. Introduction

Teaching mathematics as a modelling tool is an important goal of the curricula in many countries. According to the constructivist theories, curricula give more and more place to the application of mathematics to solve daily life problems. These problems are usually known as “modelling problems”. In this context, functions seem to be very important modelling tools. Problems related to economy, health, demography, investigations, etc. are given in the curricula and textbooks by functional approaches.

Furthermore, modelling is an important research subject in the domain of mathematics education. For more than twenty years, many studies have been carried out concerning the concept of modelling. International studies were also organized around this concept (e.g. The 14th ICMI Study in 2004, Modelling and Applications in Mathematics Education).

Today, there are several points of view concerning the term of modelling (see, Blomhøj, 2008). For a global understanding, we can refer to the definition given by Blum and Niss (1991): “While mathematization is the process from the real model into mathematics, we use modelling or model building to mean the entire process leading from the original real problem situation to a mathematical model” (p.39).

Thus, modeling is a process that contains several stages. According to Niss (1989), Blomhøj (1993) and Gregersen & Jensen (1998), Blomhøj and Jensen (2003) define these stages as following:

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1. Formulation of a task (more or less explicit) that guides you to identify the characteristics of the perceived reality that is to be modeled.
2. Selection of the relevant objects, relations etc. from the resulting domain of inquiry, and idealization of these in order to make possible a mathematical representation.
3. Translation of these objects and relations from their initial mode of appearance to mathematics.
4. Use of mathematical methods to achieve mathematical results and conclusions.
5. Interpretation of these as results and conclusions regarding the initiating domain of inquiry.
6. Evaluation of the validity of the model by comparison with observed or predicted data or with theoretically based knowledge.

The concept of function has been also a subject of research for more than thirty years in the domain of mathematics education. Many studies, which have been carried out since, show that function is a complex concept, there are various ways to introduce it in class and students may have many misconceptions about this concept.

So, we suppose that teaching functions as modelling tools requires a specific approach. According to the literature on the concept, we define the base of such an approach with the following items:

1. In order to give to the functions a statue of modelling tool, it is necessary to introduce the concept from a qualitative approach (Rene De Cotret, 1998; Comin, 2005).
2. Various semiotic registers of representation related to the concept (numerical, graphical, algebraic...) should be taught (Duval, 1993).
3. Efficiency of framework changes to solve problems should be shown on relevant examples (Douady, 1986).
4. Reference functions (affine function ($ax+b$), square function (x^2), reverse function ($1/x$), etc.) should be studied in class and the fact that these functions make it possible to generate other functions and to deduce their properties should be shown (Erdogan, 2006).

This new approach of the functions exists today in some countries. The current mathematics curriculum of the 10th class in France constitutes, in general aspects, an example of this approach.

In this study, we are interested in first-year Turkish primary teacher education students' ability to use functions to solve problems containing a modelling dimension. Students who attend national exams at the end of the high school and who want to choose this department have to do two math tests. In these exams, maths and Turkish are the most important tests for these students. Math tests contain questions from number sets to calculus course. Thus, at the beginning of the university these students know, in theory, the concept of function, some types of functions and their applications. So, we suppose that an approach of modelling for the concept of function constitutes an essential component of these primary teacher education students' mathematical knowledge.

2. Method

This study was conducted with 51 first-year primary teacher education students at the Anadolu University during the 2008-2009 academic years. A questionnaire composed of 4 problems was suggested to the students. It was carried out at the end of a qualitative teaching of the functions. This teaching primarily dealt with the concept of function, reference functions and their applications. It lasted about three months at the rate of two hours per week. In the questionnaire, we asked students to answer if it is possible to solve these problems by using functions. Students were not expected to completely solve these problems but only to indicate if it is possible and how.

2.1. Presentation of the problems

Problem 1: Two telephone companies offer different prices. One minute of communication costs 0.25 euro with the first company while the second company asks for 8 euro for a monthly use and offers an attractive price of 0.17 euro per minute. How many minutes at least should a user speak per month to choose the second company with advantage?

This problem can be easily solved by setting up the equation $8+0.17x=0.25x$. The answer is 100 minutes of communication. From a functional approach, the problem can be solved using the graphs of the affine functions $f(x)=0.25x$ and $g(x)=0.17x+8$. With the graphs drawn in the same plane, it is easy to see that starting from value 100, the graph of the function f is above the graph of the function g . However, an algebraic or numerical work could be necessary to find the intersection point of two graphs.

Problem 2: A manufacturer wants to produce lots of tins of one litre while using minimum metal. What should the dimensions of a tin be?

This is an optimization problem. A tin is cylindrical and the volume of a cylinder of height h and of radius R is given by the formula $V = \pi R^2 h$. The formula of the metal area, which is necessary to construct a tin, is $A = 2\pi R h + 2\pi R^2$. This is the area of two circles (lids of the tin) and the area of the rectangle. As the volume of the tin is $\pi R^2 h = 1000$ ($1000\text{cm}^3 = 1\text{dm}^3$), $h = 1000 / \pi R^2$. So, $A = 2\pi R^2 + 2000 / R$. ψ is a function of the radius

This question can be solved by the derivative, but an algebraic or functional solution is more difficult. For an algebraic solution, it is necessary to seek the minimum of the function and in which point it is reached. The minimum of this function can be also identified, roughly, on the graph. But, as we only asked if it is possible to use functions, it was enough to explain that the metal area can be expressed by a function of the radius and this function should have a minimum for a value of the radius.

Problem 3: Ahmet’s home is 8km away from his school. This morning, Ahmet decided to go to school on foot. He walked 4 km in half an hour. But after he got tired and his speed decreased by half for each half following hour. How much time would Ahmet have spent this morning to arrive at his school?

This problem is related to the concept of sequence and the concept of limit. But the distance-time graph indicates a decreasing function and the sequence of the numbers $4, 4+2, 4+2+1, 4+2+1+1/2$ indicates an increasing function which only reaches the value 8 within the meaning of limit.

Problem 4: A cylindrical pitcher of 2cm radius and 10cm height can be filled up by a cylindrical glass of 1 cm radius and 5cm height. How many glasses of water does the pitcher contain when it’s full?

After calculating the volume of the pitcher and the volume of the glass, the solution can be easily found. Thus, it is neither necessary nor useful to use functions. We put this problem to make sure if students are able to make a distinction between a modelling problem for which we use functions and a problem which is solved algebraically.

2.2. Analysis of students’ answers

From a theoretical point of view, each problem above can be solved by various *techniques* (Chevallard, 1997). We identified six of them (algebraic, functional, algebraic - functional, numerical, arithmetic-numerical, analytical). We classified students’ answers according to these techniques. In this classification, we took two types of answers into account. The first one (the first three columns in the tables below) is composed of the answers which do not answer the question. In this case, students generally seek a solution without indicating if the use of functions is possible. The second one (the five following columns in the tables below) is composed of the answers which initially answer the question about the possible use of functions. Answers in this category mainly consist of explanations. However, it was not possible for us to classify some answers because they were either very short explanations or very incomplete calculations. These answers are noted in the tables below as other answers.

3. Results

Table 1. Students’s answers to the first problem

Techniques / answers	38 answers and 13 non answers					Answer to the question				Total
	Non answer to the question			Wrong Response		Correct Explanation		Erroneous Explanation	Incomplete Explanation	
Algebraic	5	4	0	0	0	0	1	4	14	
Functional	0	0	1	0	0	0	7	1	9	
Algr-funct.	0	1	1	0	0	0	0	0	2	
Numerical	1	2	0	1	0	0	3	0	7	
Arith-num.	0	0	0	0	0	0	0	1	1	
Analytical	0	0	0	0	0	0	1	0	1	
Others	-	-	-	-	2	-	1	1	4	
Total	6	7	2	1	2	0	13	7	38	

15 students immediately start a solution without answering to the question. 9 of them seek an algebraic solution. But only 5 of them obtain a correct solution. In the wrong solutions, we notice that the lack of use of graph becomes

a problem. 22 students declare that it is possible to use functions. But no student succeeds in giving a correct explanation. These students never use the graphs of two affine functions. The underlined request “by using functions” seems to have led these students to seek a domain and a range for algebraic expressions $0.17x+8$ and $0.25x$ in order to be sure that each expression represents a function within the meaning of correspondence between two sets.

Table 2. Students’s answers to the second problem

Problem 2 Techniques /answers	38 answers and 13 non answers					Answer to the question			
	Non answer to the question					Correct	Erroneous	Incomplete	Total
	Correct Solution	Wrong Solution	Incomplete Solution	Wrong Answer	Non Explanation	Explanation	Explanation	Explanation	
Algebraic	0	1	1	2	0	0	0	1	5
Functional	0	2	0	5	0	0	2	3	12
Algr-funct.	0	0	0	0	0	0	3	1	4
Numerical	0	2	0	0	0	0	3	0	5
Arith-num.	0	0	0	0	0	0	0	0	0
Analytical	0	4	1	0	0	0	1	1	7
Others	-	-	-	5	0	-	-	-	5
Total	0	9	2	12	0	0	9	6	38

12 students think that it is not possible to solve this problem by using functions. 15 students affirm that it is possible but they give an erroneous or incomplete explanation. 5 students start a solution with an analytical approach. In these solutions, students explain that it is possible to solve the problem by the derivative of the metal area but they do not note any relation between the concept of derivative and the concept of function. Some students who think that it is not possible to use functions explain that such a situation cannot be represented by a function. They explain that the radius and the height are not given or unknown, so it is not possible to express the problem by a function. For some of other students, there could be various functions according to the radius and the height, so it is not possible to use functions. And for a few students, all necessary information to define a function is not given, e.g. the range. All these explanations show that, for these students, when there are unknown or unspecified values in a problem it is not possible to represent it by a function. On the other hand, students who think that it is possible to use functions mention some important concepts like graph, variable, maximum, minimum, etc., but they do not succeed in giving an acceptable explanation.

Table 3. Students’s answers to the third problem

Problem 3 Techniques /answers	45 answers and 6 non answers					Answers to the question			
	Non answers to the question					Correct	Erroneous	Incomplete	Total
	Correct Solution	Wrong Solution	Incomplete Solution	Wrong Answer	Non Explanation	Explanation	Explanation	Explanation	
Algebraic	0	0	2	0	0	0	2	0	4
Functional	0	0	1	2	0	0	4	10	17
Algr-funct.	0	1	1	0	0	0	1	0	3
Numerical	0	1	8	4	0	0	0	6	19
Arith-num.	0	0	1	0	0	0	0	0	1
Analytical	0	0	1	0	0	0	0	0	1
Others	-	-	-	-	-	-	-	-	-
Total	0	2	14	6	0	0	7	16	45

17 students give an answer within the functional framework and 19 others within the numerical framework. The numerical answers often consist in a calculation of the values by the 30 minutes intervals. In this technique, students who give an erroneous answer often do not lead their calculations until infinitely small values and stop calculating on a rank. When we look at the 14 erroneous or incomplete explanations within the functional framework, we notice that these students explain the fact that the distance depends on speed and some of them show this relation by the formula $Distance = Speed \times Time$. Consequently, they think that this situation can be represented by a function.

Some students indicate that it is possible to use graphs but they do not refer, for example, to a monotonous function. Some of others give explanations within the meaning of correspondence between two sets.

Students' answers to the fourth problem are definitely better than the others. 17 students solve this problem correctly and 12 others give a correct explanation. However, 13 students think that it is possible to solve this problem by using functions. These students also seek to define a function within the meaning of correspondence.

4. Conclusion and Recommendation

Our analyses show that students have considerable difficulties to perceive functions as tools to solve modeling problems. For these students, the concept of function only appears to be a theoretical concept. The fact that students constantly try to decide if the situation can be represented by a function within the meaning of correspondence between two sets shows that they did not acquire the meaning of variation. Furthermore, students do not seem to have acquired qualitative properties of the functions and do not seem to have understood the need to study reference functions.

This result could be explained by two reasons. Initially, the way by which the concept of function is introduced at the high school seems to become an obstacle, because the concept is only introduced from a set approach at the high school. So, students perceive the concept like a rule which associates a unique element of a second set with each element of a given set. Questions in the national exams also reinforce this understanding of the concept. Secondly, the study of the functions in class does not seem to have contributed to the acquisition of the meaning of the concept and ensured a relevant field of application to the reference functions. Our hypothesis is that this study was not enough to make clear complex relations between a qualitative approach, various semiotic registers and the properties of reference functions.

We suppose that a qualitative approach of the functions and reference functions should be taught at the high school. In this purpose, the curriculum should rather attach a great importance to the tool aspect of the concept to solve problems than to its theoretical aspect.

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