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How does a taxi driver use geometry?

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Abstract

Euclidean geometry, which is often covered in mathematics curriculum in primary and secondary education, fails to meet all the requirements in daily life. For example, a taxi driver cannot use Euclidean geometry in cities using a grid city plan At this point, what method should be used because distance 'as the crow flies' cannot be used? The solution of this problem requires the use of a non-Euclidean geometry, Taxicab geometry. The aim of this study is to raise awareness of non-Euclidean geometries by means of activities comprehensible to secondary education students, who are considered to have made a certain progress in terms of level of geometric thinking.

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Keywords: Taxicab geometry, Euclidean geometry, secondary education students.

1. Introduction

Although no other type of geometry except for Euclidean geometry hadn't been taught until the 19th century, geometry showed a huge improvement in this century, like other academic disciplines. Geometry developed more than estimated due to its scope and division into different branches. As a result of this, Euclidean geometry, the only type of geometry taught in previous centuries, became a sub-field of large mathematical theories of space. Euclidean geometry, which is often covered in mathematics curriculum in primary and secondary education, fails to meet all the requirements in daily life. There are currently more than 50 types of geometry (Malkevitch 1991). This diversity indicates the richness of modern geometry, but it also forces curriculum organizers to answer the question "Which subjects and concepts of geometry should be covered in school curricula from pre-school education to higher education, and which level should they be included at?" (Ministry of National Education, 2010). Focusing on this situation with a striking illustration, this study tries to point out to the need for non-Euclidean geometries. For example, a taxi driver cannot use Euclidean geometry in cities using a grid city plan that consists of horizontal and vertical roads. At this point ,what method should be used because distance 'as the crow flies' cannot be used? The

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solution of this problem requires the use of a non-Euclidean geometry, Taxicab geometry. Transportation becomes easy using the distance function defined in Taxicab geometry. The aim of this study is to raise awareness of non-Euclidean geometries by means of activities comprehensible to secondary education students, who are considered to have made a certain progress in terms of level of geometric thinking. Although Taxicab geometry is very similar to Euclidean geometry, the concept of distance is defined in a different way in Taxicab geometry. This study aims to introduce the activities designed for secondary education students using this feature of Taxicab geometry and to come up with suggestions for teachers.

2. Taxicab Geometry with Activities

Before giving a definition of Taxicab geometry, secondary school students can be involved in an introductory activity. Students can be assigned the role of a taxi driver and they can explore Taxicab geometry by means of the relationship between the subject and daily life. In this way, they can be introduced to a type of geometry different from Euclidean geometry and, at the same time, they can see its real life implications. The following activities can be used as sample exercises that students can do.

Activity 1: How does a taxi driver use geometry?

Imagine that you are a taxi driver in Savannah, Georgia and its street map is given below. At the end of the day, you are supposed to fill in the daily log that your boss gives to you. Your boss asks you to pick up and drop off passengers 5 times throughout the day.

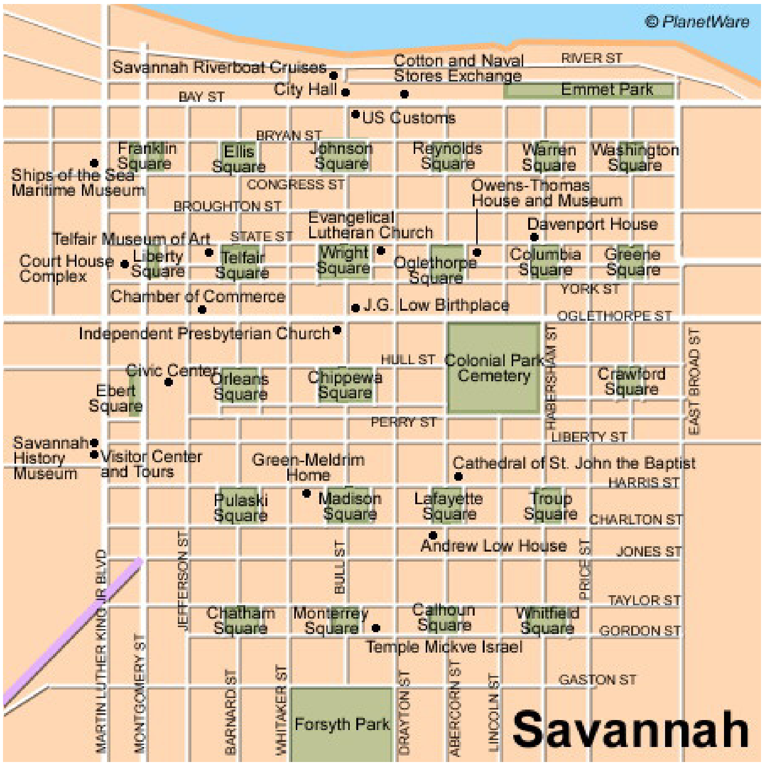

Figure 1: Street map of Savannah city

1. For this activity, you will use the map given in Figure 1. You will decide where to pick up and drop off a passenger, but you should determine different routes for each of the five journeys. The stops (i.e. the places to pick up and drop off passengers) are indicated by black points on the map.

2. You should determine the stops to pick up and drop off a passenger and write them in the daily log (see Table 1). You should make sure that the route you will choose is the shortest one.

Table 1: The daily log in Taxicab Geometry

Rounds	Passenger Pick-up Points	Passenger Drop-off points	Measure of the streets (Taxicab Geometry) km
1-Blue			
2-Green			
3- Red			
4- Orange			
5- Purple			
			Total km:

3. You should write the number of journeys and draw your route on the map in different colors for each round (see Figure 2 for an example). After drawing a route on the map, you should measure the journey distances using a ruler for each round and write it in the daily log.

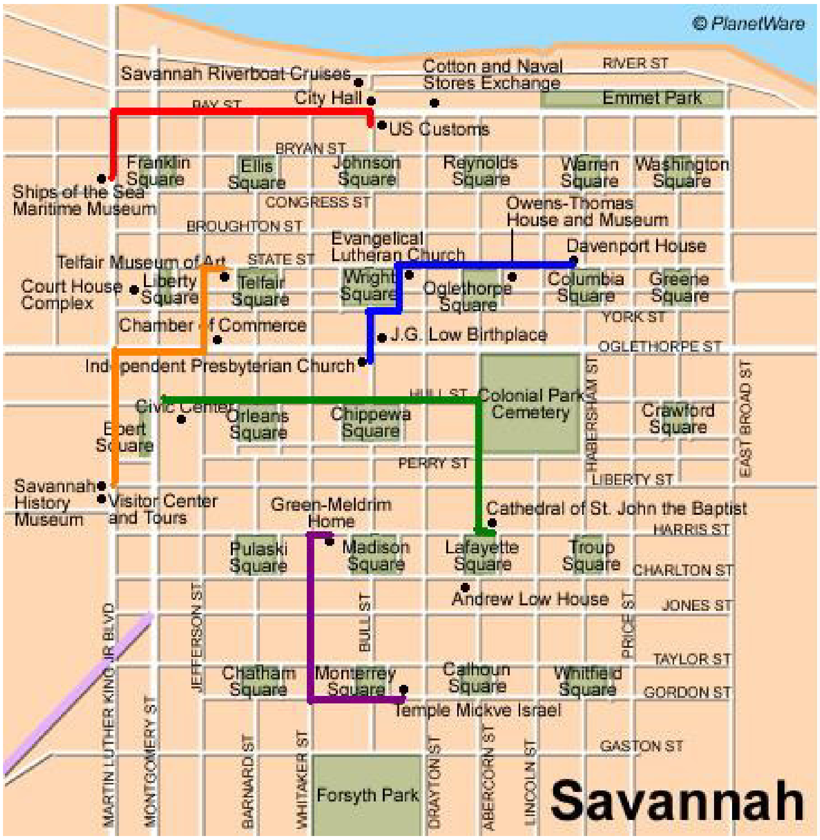

Figure 2: Sample routes for 5 rounds

4. Since 1cm on the map represents 100 m, you should specify the distance of each round in the daily log.
5. You should state the total distance of the journeys for the day.
6. After performing the first five steps, measure the least distance between the Passenger Pick-up and Passenger Drop-off points based on the straight line, or "as the crow flies", (Euclidean Distance). Measure this for each round and fill in the daily log(see Table 2).

Table 2: The daily log in Euclidean Geometry

Rounds	Passenger Points	Pick-up	Passenger points	Drop-off	Measure of the crow flies (Euclidean Geometry) km
1-Blue					
2-Green					
3- Red					
4- Orange					
5- Purple					
					Total km:

7. Compare the results obtained from the two tables (Table 1 and Table 2).

a) What is the sum of the distances when the taxi driver follows the routes between the passenger pick-up and drop-off points for each of the rounds determined? _________ How many km are the distances measured "as the crow flies"? _________

b) Compare the sum of the distances when the taxi driver follows the routes and the sum of the distances measured "as the crow flies" _______

c) Does the taxi driver cover more distance when he or she follows the routes determined or the routes determined "as the crow flies" (Euclidean Distance)?

By looking at the tables, can we suggest that the distance between two points measured "as the crow flies" (Euclidean Distance) is always more than the distance covered when the taxi driver follows the routes? If yes, why? If no, why?

By means of this activity, students get the opportunity to measure the distance between two points in both Euclidean geometry and in Taxicab geometry without knowing the definition of it.

Taxicab geometry is quite different from Euclidean geometry, which most of us know. In Taxicab geometry, it is possible to move through mutually perpendicular horizontal and vertical lines when moving between two points. Imagine that you are a taxi driver in a city with a grid street plan. The taxi driver only follows the roads. He or she just drives straight or can turn left or right at a right angle. As two points are connected with a straight line in Euclidean geometry while two points are connected with the routes followed by a taxicab, the latter type of geometry is named Taxicab geometry.

As seen in Figure 3, a taxi driver must use horizontal and vertical lines going from point A to point B unless he or she has wings and can fly. Therefore, because the route from A to B would not be blue, it would be more than $\sqrt{41}$. The taxi driver can follow the red, yellow or green routes from A to B, but not the blue route. In this case, which one is shorter: the red, yellow or green one? As you can see, all the three routes are nine blocks away (see figure 3).

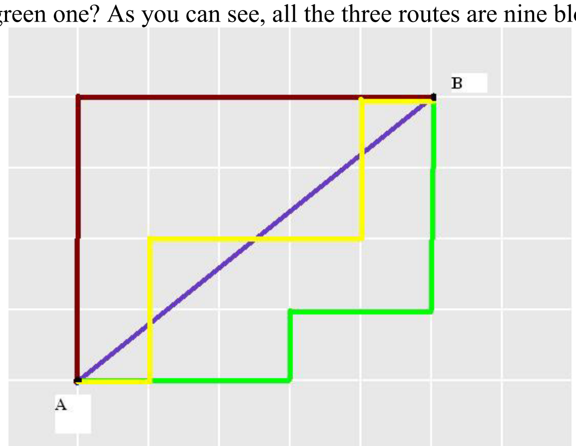

Figure 3: Sample routes for round five

If we find "the distance formula" for the taxi driver considering the other possible starting points and destinations, we can conclude that the taxi driver uses $d_T(A,B) = |x_B - x_A| + |y_B - y_A|$
to measure the distance between $A(x_A, y_A)$ and $B(x_B, y_B)$.

d) If we know the taxicab distance between two points, can we measure the Euclidean distance between these points?

This step aims to have students realize that it is necessary to add together the lengths of two vertical edges of a right triangle in Taxicab geometry and it is necessary to calculate the hypotenuse of the right triangle in Euclidean geometry. In this way, as a part of "Geometry and Measuring" subject in Grade 8, students "learn about the Pythagorean relation, they can solve related problems, they can perform real-life applications of the Pythagorean relation, and they can find the distance between two given points on the coordinate plane using the Pythagorean relation". By means of these activities, they are also introduced to Taxicab geometry, which is a type of non-Euclidean geometries.

Activity 2: How does a ropeway driver use geometry?

A new transportation project in Ankara city plans to reduce travel time to 13.5 minutes by ropeway transportation from 25-30 minutes by bus or private vehicles. According to this project, the first stage of the ropeway transportation system has three stations and alternative station points are given so that locations of the stations can be determined accurately (i.e. the shortest distance in terms of cost and time. Which of the points given in Figure 4 could be the correct locations? Explain the reasons for your choice.

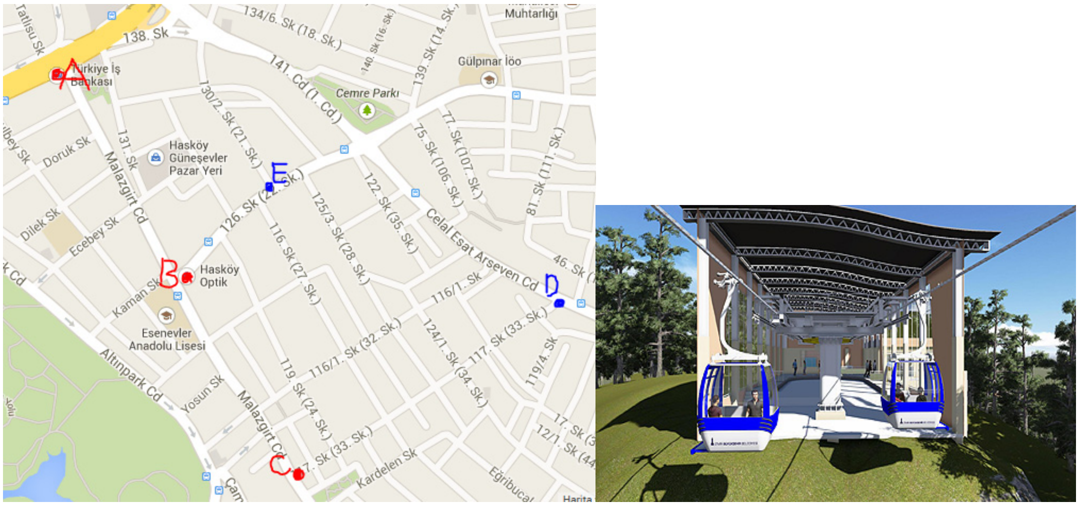

Figure 4: Location of alternative station points of ropeway transportation system

By means of this activity, students will discover that Euclidean distance is favorable when they measure the distances between the stations that are not on the same route in the map given in Figure 4. When they use the existing routes, on the other hand, they will discover that Taxicab distance is more convenient. They will realize that the Taxicab distance and Euclidean distance between the three routes on the same route are equal. For example, the Taxicab distance and Euclidean distance between the stations A, B, and C on Malazgirt Street are equal. However, because there are no roads between the stations A, E, and D station, the Taxicab distance and Euclidean distance between them are not equal (see Figure 4). Therefore, locating the ropeway transportation stations on a route covering the points A, E, and D is favorable. In this way, while determining the three stations, students will compare the distances by measuring the Taxicab distance and Euclidean distance on the grid plan (i.e. a graph consisting of horizontal and vertical lines) of the city. As a result of this comparison, they will finally discover that Euclidean distance is more advantageous in terms of the distance between the points on non-horizontal or non-vertical (curved) lines.

Taxicab geometry, which everyone meets and uses unconsciously most of the time, has many uses in everyday life. In Taxicab geometry, many basic concepts that we are familiar with in geometry are different from Euclidean geometry (e.g. circles and squares).

Activity 3: Differences between Taxicab geometry and Euclidean geometry

A convention is held in a city with a grid plan (see Figure 5). In order to make sure that transportation is convenient, guests will stay at hotels at certain distances from the convention center.

Figure 5: The convention grid plan

Your task:

Find the number of the hotels located five or less blocks away from the convention center?

What is the locus of the points five blocks away from the convention center?

In Figure 6, the yellow point represents the convention center and the blue spots indicate the hotels. Students are expected to assume that the guests will get to the convention center by taxicabs or on foot, use the area within five blocks away from the center and the horizontal and vertical roads (Taxicab distance) on the map, and obtain the red square in Figure 6 using the ruler. In this case, this square actually indicates a circle in Taxicab geometry (Ada& Kurtuluş, 2012).

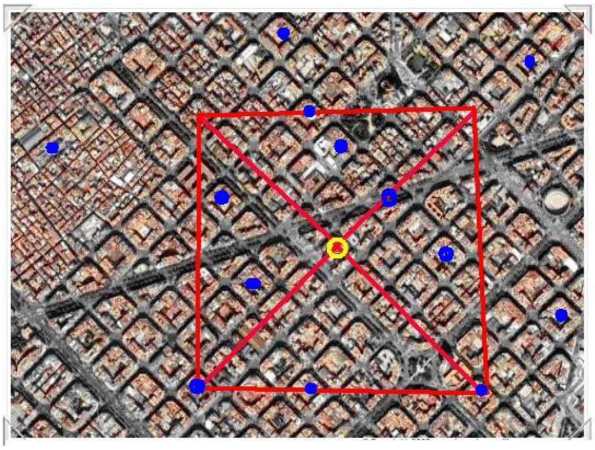

Figure 6: Points that are five blocks away from the convention center

The aim of this activity is to remind students that points at equal distances from a particular point indicate a circle and to have them discover that circle is actually a square they obtain in Figure 6 in Taxicab geometry.

Since this city has a grid plan, the roads are mutually perpendicular horizontal or vertical lines. When we consider the city grid plan on a vertical coordinate system with its origin as the convention center, the quadrilateral in Figure 6 corresponds to the square in Figure 7 in rectangular coordinate system.

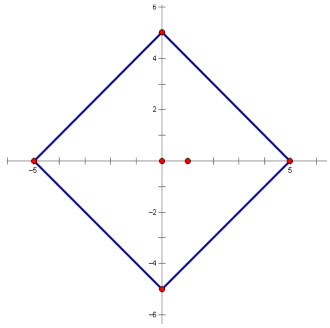

Figure 7: Points that are five blocks away from the convention center

At this stage, the question that may come to mind is, "Does the square in Figure 6 correspond to a circle in Taxicab geometry?" If this shape is to create a taxi circle, it should conform to the definition of circle. When the circle formed by the points that are five units away from the center (0,0) is considered, the point (5,0) is five units away from the center and the point (5,5) on this shape is 10 units away from the center. The point (5,5) cannot be on the circle according to the definition of circle. Therefore, a square located horizontally in the coordinate plane cannot be a taxi circle. What is the difference between the squares in "Figure 7 and Figure 8"? Although both of them are squares in Euclidean geometry, the square in Figure 7 indicates a circle in Taxicab geometry, but the square in Figure 8 indicates a square in Taxicab geometry (Janssen, 2007).

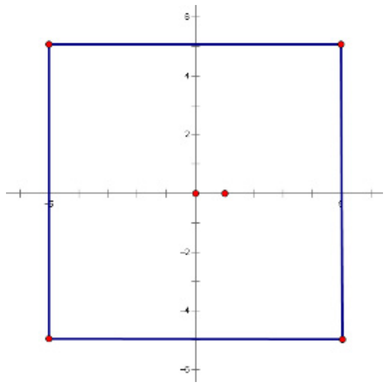

Figure 8: A square in Taxicab geometry and Euclidean geometry

By means of this activity, students will recognize some of the differences between Euclidean geometry and Taxicab geometry.

3.Conclusion

Menger (1952) described Taxicab geometry saying, "You will like geometry. Don't say that it is impossible. You can say that geometry is boring. It has become stereotyped and dead for centuries, but you are wrong. Geometry is fun, genius, beautiful and deep and very important. Geometry is alive and it is developing day by day. Just follow the development of the geometric world of planar shapes over centuries". The aim of this study, which is based on the question "how does a taxi driver use geometry?", is to have secondary school students explore Taxicab geometry. Research suggests that learning Taxicab geometry improves students' problem solving skills and helps them better understand Euclidean geometry (Janssen, 2007; Ada & Kurtuluş, 2012; Ada, 2013; Milner, 2007; Fout, Marker, Lotz & Porter, 2012). However, the literature about this subject is mostly about high school and university level. On the other hand, secondary school mathematics curriculum aims to improve students' problem solving, reasoning, mathematical communication and linking skills. In fact, introduction of Euclidean geometry to students at an early age will help them develop these skills at an early age.

As can be seen in the activities, we could suggest that Taxicab geometry is one of the most appropriate geometries among non-Euclidean geometries. Also, it is recommended that secondary school mathematics curriculum include activities revealing the role of mathematics in our daily lives. Taxicab geometry supports curriculum with this aspect as well. According to the mathematics curriculum, students in Grade 7 "are provided with activities that help them relate determining locations in the coordinate system and real life situations". As a part of geometry and measuring subject, Grade 8 students "are provided with activities about real life applications of the Pythagorean relation and activities about finding the distance between two points given in the coordinate system by using the Pythagorean relation."(MEB, 2013). In the light of this, students could be shown that the sum of the legs of a right triangle gives taxi distance and the hypotenuse gives Euclidean distance, and this situation could be linked to the Pythagorean relation. Secondary school teachers could promote students' motivation about problem solving skills by covering these activities in Taxicab geometry, which provide students with real life applications in teaching geometry and define the distance between two points in a different way from Euclidean geometry, in their lessons. In addition, by means of introducing Taxicab geometry, which is a non-Euclidean geometry, teachers can have students compare it with Euclidean geometry. In this way, covering these activities in teaching-learning process could help students have a better understanding of the structure of Euclidean geometry.

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