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Transfer Skills of Middle School Pre-service Mathematics Teachers from Informal to Formal Mathematical Language: Turkey and United States Cases*

Ortaokul Matematik Öğretmen Adaylarının İnformalden Formal Matematik Diline Çevirme Becerileri: Türkiye ve Amerika Birleşik Devletleri Durumları

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ABSTRACT: Mathematics, as universal language, is used by individuals with different native in a way that is influenced by their native languages. Two kinds of uses, formal and informal, of mathematical language arise since the mathematical statements or sentences can be expressed in a native language by defining the meaning of symbols in that native language without the use of the symbols themselves. Translation from a certain native language to the language of mathematics can be regarded both as transforming sentences from native language to mathematics and, more general, as transforming real-life events to mathematical models. The aim of this study is to investigate transfer skills from informal to formal mathematical language of middle school pre-service mathematics teachers from the USA and from Turkey. In this qualitatively designed study, data were obtained through a written questionnaire and clinical interviews. Research results show that participants from Turkey had difficulties in terms of constructing quantities apart from the difficulties of not recognizing the functional relationship in word problems. On the other hand, the participants from USA had difficulties with transferring informal sentences and statements to formal mathematical language.

Keywords: Mathematical Language, Pre-service Mathematics Teachers, Mathematizing

ÖZ: Evrensel bir dil olan matematik dili farklı uluslar tarafından farklı anadil desteği ile kullanılır. Matematiksel ifade ya da cümleler sembol kullanmaksızın anadilde de ifade edilebildiğinden, bu dilin formal ve informal olmak üzere iki tür kullanımı doğmaktadır. Bir anadilden matematik diline dönüştürme, cümlelerin anadilden matematik diline dönüştürülmesinin ötesinde genel olarak günlük yaşam durumlarının matematiksel modellere dönüştürülmesi olarak düşünülebilir. Bu çalışmada iki farklı ülkeden ortaokul matematik öğretmen adaylarının informal dilden matematik diline dönüştürme becerilerinin incelenmesi amaçlanmıştır. Nitel olarak desenlenmiş olan çalışmanın verileri yazılı soru formu ve klinik görüşmeler yolu ile toplanmıştır. Araştırmada elde edilen bulgular Türkiye'den katılan ortaokul matematik öğretmen adaylarının problemde nicelikleri ve fonksiyonel ilişkiyi oluşturma ve tanımada, Amerike Birleşik Devletlerinden katılan ortaokul matematik öğretmen adaylarının ise cümle ve ifadeleri informalden formal matematik diline dönüştürmede güçlük yaşadıklarını göstermiştir.

Anahtar sözcükler: Matematik Dili, Matematik Öğretmen Adayları, Matematikleştirme

1. INTRODUCTION

Galileo once said that "Mathematics is the language with which God has written the universe," suggesting that mathematics is the sole universal language of cosmos. Sfard et al. (1998) indicate that all should accept the universality of mathematics as a language. Mathematics has a very specific vocabulary with a syntactical and rhetorical structure like any other language and the most important characteristics making it superior is its precision. Sealey, Deshler and Hazen (2014) emphasize that the language of mathematics should be precise unlike the natural languages. Zazkis (2000) states that there is no verbosity in mathematical language

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and the mathematical nomenclature is on minimal level. Adams (2003) states that in mathematical language words speak, numbers listen, and the symbols show.

The most distinctive characteristic of mathematical language is that is used by individuals with different native languages by supporting these languages. Hence what makes mathematics a universal language is that individuals use mathematical symbols in the same notation and meaning with the support of different native languages. Statements and sentences in mathematics can be totally symbolic, but there are cases where symbols can be used together in succession with the words of a native language. Two kinds of uses, formal and informal, of mathematical language arise since the mathematical statements or sentences can be expressed in a native language by defining the meaning of symbols in that native language without the use of symbols. Zazkis (2000) claims that translation from a certain native language to the language of mathematics is in a way similar to transforming word problems to equations or, in more general terms, real-life events to mathematical models. Zazkis addresses this kind of transformation from Adler's (1998) code-switching perspective. As Zazkis points out, Adler (1998) defines code-switching as the use of multiple languages in a single conversation that are altering. According to Zazkis, even though code-switching is typical of multilingual contexts, it can also be used in monolingual mathematical classes between mathematical and daily languages. Zazkis attempted to study the idea of code-switching between informal and formal mathematical language used by students. She emphasized that she could not see the correct use of formal mathematical language in the absence of sufficient understanding. Selden and Selden (1995), regard the process of translating informal mathematical statements to formal mathematical statements as unpacking.

When mathematics is approached from a linguistic perspective and the role of a teacher is taken into consideration, the mathematical linguistic skills of mathematics teachers and their pedagogical approaches for developing students' mathematical linguistic skills gain high importance. The literature emphasizes on the necessity of teachers offering support to students from transition of daily language to mathematical register (e.g. Adams, 2003; Pimm 1987; Schleppegrell, 2007). The literature also emphasizes on the fact that mathematical language can be developed simultaneously with mathematical concepts. For instance, Pimm (1987) states that mathematical language is precise not in itself but in the way it is being used. Pimm also emphasizes that technical mathematical language can be developed through mathematical concepts. Specific responsibilities of mathematics teachers required for the development of students' mathematical linguistic skills during development process of mathematical concepts could be considered in two aspects. First is the mathematical linguistic skill, which a mathematics teacher should possess. Schleppegrell (2007) states that mathematics teachers should be conscious of the linguistic issues in both teaching and learning. Barwell (2005) states that informal language is a scaffold for formal mathematical language and emphasizes on the fact that math teachers should have the skills of using formal mathematical language on various levels. Lemke (2003) states that mathematics teachers should be able to translate between formal and informal mathematical languages as well. However, as seen in the literature, most mathematics teachers focus on the content and believe students would learn mathematical language by exposure. According to Gray (2004) the reason for such teachers' belief is that most of them are not aware how to teach mathematical language or do not believe they are capable of doing it.

Similarly, Schleppegrell (2007) also emphasizes on the fact that the students could not acquire the required formal mathematical language by being exposed to its use by the teacher or the use of textbooks. As a result, the second responsibility of teachers manifests itself as the pedagogical skills they should have for improving students' mathematical language skills. In this sense, development of mathematical language skills should be taken into consideration in terms of developing communication skills in a certain language, meaning the development of written and oral communication in that certain language. During the recent years, a significant number of developed countries have started to put significant great emphasis on mathematical language communication into the curriculum. This emphasis should start from early years of education. Also, criteria of sufficiency in mathematical language should be added to the standards in mathematics teacher's education. Veel (1999) points out to a distinctive division between written language and mathematical speech, emphasizing on the fact that a mathematics teacher's use of speech adds to mathematical interpretation on the written sources. He emphasizes on the fact that this kind of interpretation and the interaction between teacher and students' in a classroom is of vital importance in terms of mathematical education. Similarly, the importance of communication in mathematical language is emphasized in NCTM (NCTM 2000) standards by means of sharing of mathematical thinking with teachers and peers in a consistent way. Sfard et al. (1998) emphasizes on the consensus of a need to foster students' ability to 'talk mathematics'. She points out that a teacher's talking for talk's sake in the classroom is of no value and instead he/she should lecture and direct the classroom discussions in a way so as to support the process of learning mathematics. O'Halloran (2000) stresses that mathematics teachers' interpretation of the meaning of mathematical symbols is as important as their use of mathematical language when unpacking from informal to formal mathematical language.

As overemphasized in the literature, mathematics teachers' teaching abilities and use of mathematical language have very important role for students in acquiring communication skills in mathematical language. In this sense, pre-service mathematics teachers should be trained how to acquire the required communication skills as well as the pedagogical skills in relation with mathematics language. This study mainly focuses on pre-service mathematics teachers' skills of using mathematical language.

1.1 Related Literature

During the recent years, many countries have started to attach a great importance to mathematical language in their curriculum and standards. Barwell (2005) defines mathematical language development as a transition from mathematical thoughts with informal language to a communication with a more standard mathematical language. Barwell also indicates that in the recent years a large number countries have included the development of mathematical language in their curriculum. Along with increase on mathematical language in curriculum, it is noticeable that the number of studies focusing on this topic has also increased significantly. Among these studies, apart from researches made in certain semiotic frameworks, there are studies focusing on the teaching of mathematical language. Jamison (2000) investigated the effect of explicit explanation related to syntactical and rhetorical structure of mathematics language on students' understanding of mathematical concepts. As a result of such explanations, he achieved the result that students can learn and use the rules of mathematics language as a tool to understand abstract mathematical concepts. Gray (2004) claims that teachers generally focus solely on mathematical concepts and ignore the mathematical language knowledge and skills in the classroom since they expect participants to learn language of mathematics skills through being exposed to them. Gray investigated the reason behind the neglect of language of mathematics in teaching and concluded that teachers are either unaware of how to teach the language of mathematics or they do not believe they can implement such language training in their classrooms. Owens (2006) questioned the main reason behind the students' reluctance to mathematics courses, when at the beginning of the school they were observed to be very eager towards mathematics. Owens emphasizes on the fact that teachers need to know at which point in the curriculum they should introduce the vocabulary and how to make connections with the students' knowledge to support the use of appropriate mathematical vocabulary in the classroom. Sealey, Deshler and Hazen (2014) wanted to develop a greater awareness of calculus students' understanding and difficulties with the mathematical language. They concluded that students' use of notation and understanding of notation were not necessarily related to understanding at this level of mathematics. Moreover, they indicated that they were surprised to find out the students' written descriptions were better than verbal descriptions. Schleppegrell (2007) states that the construction of mathematical knowledge depends more on the oral language explanations and interaction of the teacher with students compared to any other discipline; thus the teacher plays a key role in helping students learn how to negotiate the symbols, diagrams, and technical language. Furthermore, teachers should use oral language to unpack and explain the meanings to help students gain different meaning for understanding. Selden and Selden (1995) investigated the calculus students' ability to unpack informally written mathematical statements. They defined associating an informal mathematical statement, in which the logical conjunctions and quantifiers are not clearly expressed, with a consequentially equivalent statement in a formal language in which the logical conjunctions and quantifiers are clearly expressed as the unpacking of that informal statement. In short, they point out to the formal statement consequentially equivalent to the current statement with the unpacking of the abovementioned informal one. The conclusion of the study was that all students, the majority of whom were in their third or fourth-year university, specializing in mathematics or secondary mathematics education, could not consistently unpack informally written mathematical statements into equivalent formal statements. Herbel-Eisenmann (2002) focused on analyzing and understanding the relationships between covarying quantities in two eighth grade classes. She concluded that students can learn the language by exploring various contexts. Using multiple ways of talking about mathematical ideas enhanced student' learning and making algebraic ideas. Wang, Hsieh and Schmidt (2012) compared the pre-service mathematics teachers' thinking and reasoning skills on mathematical language from USA and Taiwan, who were chosen based on TEDS-M. Even though Wang and et al. concluded that Taiwanese candidates demonstrated a better performance when compared to the teachers from USA, they point out to the fact that both countries shared some weaknesses in this sense. As a result, the authors suggested that opportunities should be provided for pre-service mathematics teachers so as to improve their skills related to mathematical language.

1.2 Aim of the Study

The aim of this study is to investigate transfer skills of middle school pre-service mathematics teachers from two different countries, one from the USA and one from Turkey from informal to formal mathematical language. Research questions of the study as following:

- How are the middle school pre-service mathematics teachers' transfer skills from informal to formal mathematical language?
- How the case of differentiation of middle school pre-service mathematics teachers' transfer skills from informal to formal mathematical language in accordance with the middle school mathematics teachers training program?

2. METHOD

2.1. Participants

Eight middle school pre-service mathematics teachers from each university, one from the USA and one from Turkey, were chosen a volunteer basis among the candidates having taken most of the mathematics courses and were in their fifth semester in both Universities. Throughout the text we refer to these two groups of teachers as AP1, ... AP8, and TP1, ..., TP8.

2.2. Instruments

In this qualitatively designed study, data were obtained through a written questionnaire and clinical interviews. The students were first asked to complete questionnaire on the paper prior to discussing their responses with the interviewers. For each question to which they wrote their response, the students were asked to explain it verbally.

The written questionnaire was prepared firstly in English to investigate transfer skills of participants from informal to formal mathematical language. For the content validity, clarity, simplicity, and ambiguity of the items, an expert was asked to evaluate the questionnaire. Then the instrument was translated into the Turkish and reviewed an expert in English from Turkey for ambiguity. The questionnaire had two parts. In the first part, two sentences and a statement in native language were given and the participants were asked to transfer these into formal mathematical languages. We adapted Selden and Selden's (1995) terminology of unpacking when referring to transfer of sentences or statements from informal, spoken language to formal mathematical language. Two sub-items questioning truth of the given sentences were also added to reveal the participants' understanding of informal statements in native language and to unfold the ways used for the transfer of written to mathematical language. The participants were given the following sentences in their native language and asked to write them in mathematical language.

1. "If you add two real numbers in either order you get the same result."
2. "The product of two real numbers differing by two is always one less than a perfect square."
3. "Set of numbers bigger than or equal to two."

After writing the given sentences in mathematical language, for each one of them the participants were asked the following two sub questions

- The statement is true [circle one]: always, sometimes, never
- Considering your answer about the truth of the sentence, which symbol(s) or word(s) in your expression indicates that the sentence is always, sometimes, or never true?

The first sentence can be rewritten in mathematical language as a conditional proposition either with the conjunction "if" or with the use of universal quantifier:

$$\text{“}a, b \in \mathbb{R} \Rightarrow a + b = b + a\text{”}$$

$$\text{“For each } a, b \in \mathbb{R},\ a + b = b + a\text{”}$$

The second sentence is also a conditional proposition and actually it emphasizes the existence of a perfect square. The proposition suggests that the product of two real numbers differing by two is one less than a perfect square. On the condition that if the difference of real numbers is two (hypothesis), the product of these numbers will equal to one less than perfect square (provision). Here the expression "one less than perfect square" refers to a one less than single perfect square providing the provision. This statement refers to existence of a real number whose square minus one is equal to the product of two given real numbers. Therefore, the proposition is expressed in mathematical language as follows:

$$\text{“}a, b \in \mathbb{R} \ni b = a - 2 \Rightarrow \exists c \in \mathbb{R} \ni a.b = c^2 - 1\text{”}$$

In mathematical language, the third statement could be written as:

$$\{x \in \mathbb{R} | x \geq 2\}$$

The second part of the instrument consisted of two problems, each of which had some sub-questions. The sub-questions were added to collect consistent information about the participants' thought processes in mathematizing a daily life problem.

Problem 1. First class postage with a private postal service costs \$0.31 for all letter weights through 1 ounce, plus \$0.21 for each ounce or fraction of an ounce thereafter. Each letter is required to carry one \$0.31 stamp and as many \$0.21 stamps as necessary.

(a) Graph the relationship between the number of stamps and the weight of the letter.
(b) Give a mathematical expression about total cost of mailing a letter by using symbols.
(c) Express the relationship between total cost of mailing a letter and costs of postal service, stamp in your own words.
(d) Constitute a table for the letters whose weights are 1.5 ounce, 2 ounces and 3.4 ounce.

Problem 2. Tom builds fencing from pieces of wood as shown below.

(a) In your own words describe the relationship given by the above diagrams.

(b) Identify independent and dependent variable in the relationship that you described in part (a).

(c) Write an algebraic expression that corresponds to the relationship you described in part (a).

(d) What symbol did you use for independent and dependent variables?

2.3. Analysis

The texts of the transcriptions of the audio recordings of clinical interviews and participants' responses to written questionnaire were analyzed by Miles and Huberman's (1994) three-phase qualitative data analysis method. This analysis method includes three phases that are "data reduction", "data display" and "conclusion drawing/verification". Two researchers analyzed data by coding sparely and then common themes were determined by working on analysis together. The analysis results of the researchers based on common themes were found to be consistent (Miles & Huberman, 1994).

3. RESULTS

The findings obtained from the study indicated that the participants from Turkey were better in terms of using mathematical language for the process of unpacking, composing algebraic representations and algebraic manipulations compared to participants from USA. However, same participants faced difficulties in terms of composing quantities in problems, finding relations among quantities and demonstrating accordingly in mathematical representations. Participants from USA, facing difficulties in the process of unpacking, were more successful in articulating their mathematical thoughts verbally and in the process of mathematizing the problems.

3.1. Unpacking Results

As already mentioned, participants from Turkey were observed to be more successful than participants from USA at unpacking the given sentences or station in their native language.

Most participants from Turkey were conscious of logical structures of propositions and more often were using quantifiers and connectives.

Six out of eight participants from Turkey were able to unpack the given sentence relating it to commutative property of real numbers correctly. They identified the context and used quantifier. The following is an example of such unpacking:

"$For\ \forall a, b \in \mathbb{R}, a + b = b + a$"

These participants have also realized that this proposition is always true and they indicated that they demonstrated this fact with the symbol "$\forall$" in mathematical terms. One of the remaining two participants unpacked this sentence incorrectly by representing it as a set.

$$\{a, b \in \mathbb{R} \,|\forall \boldsymbol{a} + b \wedge b + a \Longrightarrow \; a + b = b + a\}$$

In the second sentence, “The product of two real numbers differing by two is always one less than a perfect square.”, an existence is emphasized. Five out of eight Turkish participants demonstrated that they were conscious of the logical structure of this sentence but only one was able to unpack it as an existence sentence. Remaining four participants had abbreviated the proposition with algebraic manipulations to examine the truth of the proposition. These participants were able to recognize the accuracy of the statement by making algebraic manipulations on the hypothesis and were able to write the given proposition in a form composed of single variable as “$a(a-2) = (a-1)^2 - 1$". In their explanation, they stated that the proposition was correct because they reached conclusion from hypothesis by indicating that the perfect square is “$(a\ -1)^2$”. Thus, when these participants use symbols indicating context, quantifiers and connective they were able to reach an unpack sentence. However, none of these participants were able to state a complete sentence along the following lines.

For ∀a, b ∈ ℝ such that a – b =2, a. b = x^2 -1. a.b = a (a -2) = (a -1)^2 - 1”

Only one participant was aware of the logical structure of the sentence and she claimed to have provided a proof. Further she stated that she indicated truth of the sentence by “each” in the statement of “for each real *x* and *y*, when *x*-y=2”

TP7: *I have made a proof here.*

I: *Right. What did you write?*

TP7: *If x minus y equals two, meaning the difference between x and y is two, then the product of two real numbers differing by two is one less than the square of the number in between. I used x in terms of y here. This is y square plus two y's. I added one to that number and found the perfect square. The square of y plus one minus one, meaning I found a number one less of the square number. I thought I proved this question (laughs). For each x and y real numbers such that x minus y equals to 2.*

I: *How did you conclude that the statement in mathematical language is always true and which symbol shows this?*

TP7: *For each x and y real numbers when x-y = 2, this statement is always true. I showed this with “each” symbol.*

Participants from USA had difficulty in unpacking the first two propositions but they demonstrated some success in expressing their mathematical thought in their native language. Whereas these participants, apart from two, demonstrated that they had grasped the logical structures of the given propositions, they were unsuccessful in using the written representations in formal mathematical language. But they were able to use quantifiers and conjunctions in their native languages when expressing their mathematical thoughts. When unpacking the first sentence into an algebraic equation “*x*+*y*=*y*+*x*” or as “*x*+*y*=*z*, *y*+*x*=*z*”, these participants realized that this proposition is always correct and were aware of the commutative property for addition They stated that the “=” symbol indicates this proposition is always correct. Unpacking examples for the first sentence illustrated in Figure 1. When these examples compare with Turkish participant's unpacks, one of which was “For $\forall a, b \in \mathbb{R}, a + b = b + a$”, the difference become obvious.

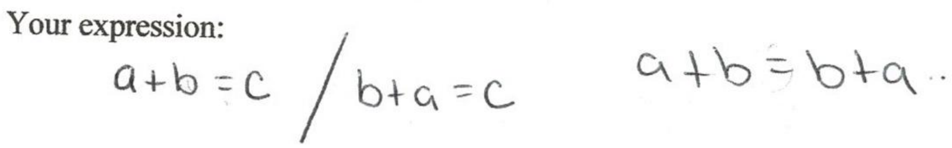

Figure 1: Example for the first sentence

One of the participants said that “there is no limitation in this statement” implying that the proposition was true for all real numbers considered as context set of the proposition in first sentence.

> ***I:*** *Okay, it should always be true. Considering your answer on the truth of the statement which symbols or a word in your expression indicates it is always true?*

> ***AP2:*** *I guess the equal sign would be the indicator. Equal sign would show this. It indicates that any limitation would be always true. Yeah. And there was no limitation in the statement. Therefore, this statement is always true.*

When examining the truth of this conditional sentence, AP2 focused solely on the correctness of conclusion in terms of the given set. For the second proposition AP2 claimed that the sentence had a limitation, implying that in the hypothesis the numbers differing by two were used and for that reason this statement was sometimes true.

> ***AP2:*** *I didn’t know if there were any symbols for a times b equals x minus one. If a and b have one of the limitations where a and b differ by two, a times b plus one equals x. That is a perfect square if and only if a times b is not equal to a negative number.*

> ***I:*** *Okay, is this always, sometimes or never true?*

> ***AP2:*** *It is sometimes true, because the statement includes a limitation where the two numbers should differ by two. Thus the limitation makes it sometimes true.*

Another participant from the USA demonstrated that he was aware of the logical structure of these propositions indicating this by meaningfully using phrases such as “any number” or “numbers differing by two”. This participant also demonstrated that he can express his mathematical thoughts in his native language clearly by using logical conjunctions such as “if…then” as well.

> ***AP4:*** *It says, “if the numbers between perfect square are one more than the answer or the product”. I said that to make sure these two numbers multiplied would differ by two. I thought if the numbers taken differ by two, then the product of these numbers will be the perfect square one more than the product. I found the number between the two numbers is squared number. I would say in general that the statement is always true.*

On the other hand, AP4 wrote down the statement “the number in between the two numbers is the squared number”. AP4’ unpack of the first statement as in Figure 2 should be consider with one of the Turkish participants’ mathematical statement, “For ∀a, b ∈ ℝ such that a – b =2, a. b = x^2 -1. a.b = a (a -2) = (a -1)^2 - 1”.

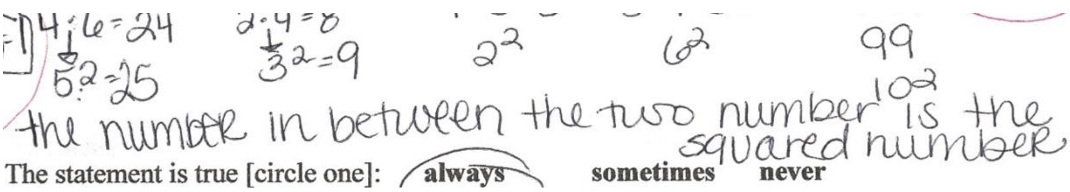

Figure 2: AP4’s unpacked the first statement

Participants from USA had more difficulty in trying to unpack the second proposition compared to the unpacking of the first statement. For instance, AP5 recalled the symbol “<” as the phase of “less than”. AP5 also used square root symbol for “perfect square”. To the contrary of interviewer’s prompts, she represented the conclusion in the second proposition as, $x(x-2)<\sqrt{y}$ inequality and was not able to reach the correct algebraic expression. The following excerpt illustrates conversation between AP5 and the interviewer in trying to resolve this issue:

> ***AP5:*** *It is true and I guess it is x times x minus two, since it differs by two and I said it is less than the perfect square. I represented it like that but I didn’t know how to represent the phrase “one less than”.*
>
> ***I:*** *Can you express that number you need to square to get one less than a perfect square?*
>
> ***AP5:*** *What would x times x minus two and one less equals to y minus one be?*

In question 3, a set was described in the statement to be unpacked. It was observed that the knowledge of participants from Turkey on set representation was very strong. It was evident that all participants from Turkey were aware of the logical structure and were able to write correctly an algebraic representation of the given set in the form $S = \{x \in R \mid x \geq 2\}$. Participants from USA, however, were unsuccessful in the set representation. Five participants were able to recall the set representation during the interview with prompt but they were not able to use this representation correctly for the given statement. However, some participants used ordered pair to represent a set. Examples from participants’ worksheets are as follows. Use of more than one variable stands out in these notations.

Figure 3: Set representative screening examples

3.2. Results of Problems

Data analysis indicates that most participants from Turkey were less successful in the process of mathematizing of problems than in unpacking mathematical statements. As for the pre-service teachers from USA it was observed they were more successful in using verbal and algebraic representations in mathematizing of problems than in unpacking process. Seven out of eight participants from Turkey had difficulties in terms of constructing quantities formation and recognizing the relationship between quantities. These participants in general had discerned a linear relationship in the problem and on the first item depicted the relationship between the number of stamps and weight of the letter with a linear graph. Furthermore, it was observed that two out of these seven participants had assumed a linear relationship between the number of stamps and weight of the letter in a reverse way of thinking. In other words, they had assumed that the weight of the letter would increase accordingly with each stamp. Three participants discerning a linear relationship had difficulties in forming quantities.

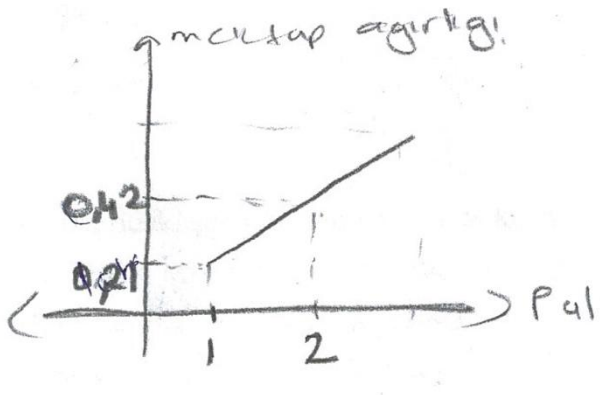

Figure 4: TP5's graph

TP5, who was confused in creating quantities, had entered price value to the number of stamp axis on the linear graph. During the interview TP5 recognized the confusion in relation with the quantity of number of stamps and tried to correct her graph as illustrated below.

> ***I:*** *How many stamps did you say are required for one and a half ounce?*
>
> ***TP5:*** *Two, one of 0,31 TL and one of 0,21 TL.*
>
> ***I:*** *All right, where will you indicate the number of stamps on graphic?*
>
> ***TP5:*** *The number of stamps will be on axis y.*

On the second item of the problem where a mathematical (symbolic) statement asked to describe the delivery cost of the letter, TP5 had proposed a mathematical statement composed of two independent variables. The participants who were not able to recognize the relationship between the weight of the letter and number of stamps came up with an algebraic equality with two variables; one for the number of stamps and the other for the weight of the letter as follows:

Total delivery cost $=0{,}31+0{,}21x+0{,}31+0{,}1y$; x: weight of the letter, y: number of stamps

Since six participants could not fully recognize the relationships between the quantities, they stated the relationship as linear in their native languages as well. For the last item of this problem, participants were asked to make a table for the letters with given weights. Five out of six had made wrong calculations in parallel to the equality they provided algebraically. The other participant who was not able to reflect on relationships in the first three items in graphical, algebraic, and native language representations had fully demonstrated correct calculations on the table. Consequently, it was considered that only one of six participants was able to recognize the relationships with particular examples but it was assumed that she was still not able to reflect on these relationships with graphical and algebraic representations. Two remaining participants from Turkey, on the other hand, were considered to be successful being able to form quantities and recognizing relationships among quantities within this question. One of these two participants was able to recognize the relationships among quantities of the problem right from the beginning and was able to recall the greatest integer function. Thereby this participant was able to make the graphic representation correctly. She was also able to state this functional relationship as a piecewise function and in algebraic form by using algebraic representation of the greatest integer function.

> ***TP7:*** *First I drew the graphic in order to show the relationship between the number of stamps and the letter: 1 stamp up to 1 ounce. 2 stamps up and equal to 2 ounces. 3 stamps up and equal to 3 ounces and goes on.*
>
> ***I:*** *Does this graphic remind you of anything?*

TP7: *It is for the greatest integer function. As for the relationships in this problem; when the weight of the letter increases as a whole number, the number of stamps will increase proportionally to the weigh and the delivery cost increases as well. If the weight of the letter is a rational number, the number of stamps will be the least whole number greater than the rational number itself. Stamp and delivery costs will be applied according to the weight of the letter.*

The other participant realized the relationship as linear like the other six participants in the instrument form. However, when the participant realized that the graph was not reflecting the relationship between the number of stamps and the weight of the letter in the interview, she redrew the graph and obtained the greatest integer function. This participant had also made the correct table representation in accordance with the graphical representation. However, TP8 obtained the algebraic statement in piecewise function form, but she could not recall the notation of greatest integer function.

TP8: *The number of stamps on horizontal axis and weight of the letter on other axis.*

I: *How did you decide that the graph should be like this?*

TP8: *If I have no stamps then the letter has no weight, thus I started it from the origin. An ounce for a stamp, two ounces for two stamps and it goes on linearly. I thought it would grow increasingly in a linear form… but when the weight of the letter is assigned 0.5 ounces there should be 1 stamp (looks at the graphic). Mmm, all right now I see that I interpreted the relationship between the numbers of stamps and cost wrong on the graphic. Ok (laughs). Yes, I misinterpreted this and drew the graphic wrong (smiles). (Redraws the graphic).*

$$T = \begin{cases} A < 1 \text{ ons}, & 0{,}31 \text{ TL} \\ A \geq 1 \text{ ons}, & 0{,}31 + 0{,}21.(1 - A) \end{cases}$$

Figure 5: TP8's algebraic representation

Most of the participants from Turkey faced difficulties with the pattern problem just as in the first problem. Only three out of eight were able to obtain an algebraic equality by generalizing the functional relationship within the problem. Two out of three were able to think functionally from the beginning to the end in the solving process of this pattern problem and obtained correct solution. Both participants thought of the total number of pieces composing the fence as a relation to the number of steps (number of diagram) and found the algebraic equality offering the generalization by focusing on the dependency relationship. The excerpts below illustrate the abovementioned reasoning processes of one of these two participants.

TP7: *The number of squares increases as the number of figures increase and the total number of pieces of wood accordingly.*

I: *What are the dependent and independent variables in this relationship?*

TP7: *Independent variables are the figure and square number. Dependent variable is the total number of pieces of wood.*

The last participant making the generalization of the problem described it first by recursive relationship and after thinking functionally explained that the total number of wood pieces changes according to the diagram number. This participant found the total number of pieces based on the diagram numbers by using table representation and wrote the algebraic equality indicating the generalization.

Even though two out of five of the remaining participants were able to express the dependent and independent variables in the pattern problem meaningfully in their native languages, they still were not able to leave the recursive thinking behind and see the functional relationship. Their algebraic equality reflects recursive thinking and does not lead to the functional relationship. These participants explained the dependent and independent variables in the problems respectively by diagram number and total number of pieces of wood; however, since they had focused on recursive thinking, they stated that independent variable x in equality "$y = 3x + 4$" was the number of wood that was added to the first figure. The statement of one of these participants is illustrated below:

> ***TP5:*** *There are four pieces in the first fence. The number of pieces increases in proportion with length of the fence. For every other fence 3 pieces are added as well.*
>
> ***I:*** *What are the dependent and independent variables in this relationship? How did you explain?*
>
> ***TP5:*** *Number of fences is the independent variable. Since we can have as many fences as we like, the total number of pieces is the dependent variable. The total number of pieces will vary depending on the number of fences.*

Participants from the USA were observed to have less difficulty with the second problem compared to the first. Six out of eight were able to recognize the relationship within the problem and demonstrated that they were able to distinguish the dependent and independent variables and reach a generalization. The insufficiency observed in these participants during the process of pattern generalization was that there was a tendency to represent the generalization with algebraic statement without the use of dependent variable. Even though these six participants had started the pattern problem with recursive thinking, they were able to express the relationship, dependent and independent variables clearly in their native languages and they were able to generalize the pattern as well. In short, these participants were able to recognize the functional relationship within the problem. Statements of one of those six participants depicting the variables explicitly are as follows:

> ***AP6:*** *I said each diagram increases by three pieces of wood.*
>
> ***I:*** *Okay. Identify independent and dependent variables.*
>
> ***AP6:*** *Independent variable was the diagram number and dependent was the total wood pieces.*

As seen in Figure 6, AP6 had also generalized the pattern as "3n+1" in the written form in the worksheet where n stands for the independent variable. However, she did not mention the dependent variable neither in her native language nor in symbolic form. During the interview, AP6 indicated the dependent variable by t with prompt and reached the equality.

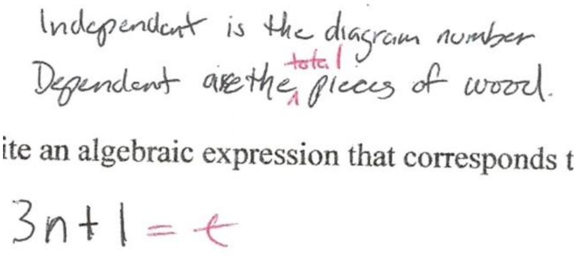

Figure 6: AP6's study paper

The other five participants had thought of the problem recursively and they were aware of the functional relationship within the problem; they did not assign a symbol to the term "the total pieces of wood" defined as the dependent variable in their native languages. These participants were able to express the relationship within the problem in their native languages, but they were not able to show this in algebraic equality. They were only focused on finding an algebraic statement for the generalization. Just like AP6, these participants too assigned a symbol to the dependent variable and were only able to write the algebraic equality with interviewer's guidance. One of these participants realized this void in the process and completed it without the guidance of the interviewer as can be seen from the excerpt below:

> ***AP7:*** *The total number of pieces of wood increases by three each time. So when describing the relationship, I said that the dependent variable would be the total number of woods. It depends on what the diagram would give us as the total number of woods. Yes. Therefore, independent variable would be diagram number. And also the diagram number representing independent variable should be positive.*
>
> ***I:*** *What would x be?*
>
> ***AP7:*** *Diagram number. The number of the diagram one, you add one,*
>
> ***I:*** *Then what do you have?*
>
> ***AP7:*** *Oh… this equals to the total number of woods. I forgot to represent the total number of woods in my algebraic expression. Yeah, I'll use y for the total of number of wood pieces representing the dependent variable. (Laughs).*

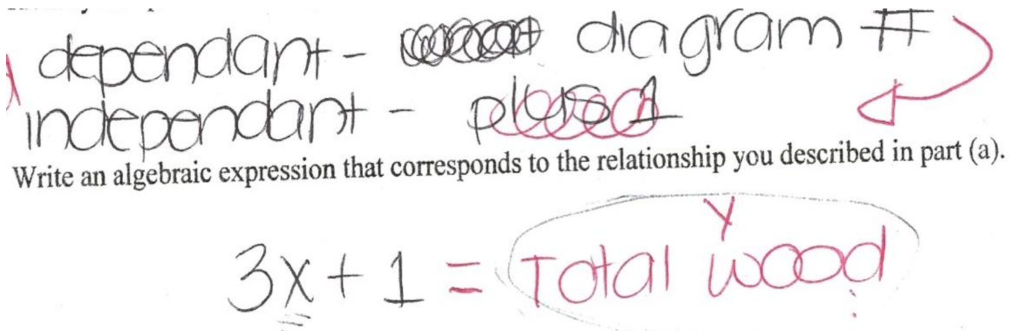

Figure 7. AP7's paper

Two remaining participants, however, were the only ones not being able to develop functional thinking in the pattern problem. They approached the problem fully from recursive perspective and were not able to define the dependent and independent variables. Thus, they were not able to express the relationships as algebraic statements in the form “3x+4” and “n+3”. Excerpt from the interview one of these participants is as follows:

> ***AP8:*** *Tom continuously adds three pieces of wood meaning n plus three … there is the original post and dependent variable was in each one active. Algebraic expression that corresponds to the relationship described is n plus three, but I wrote down the original as … (thinks out loud)*
>
> ***I:*** *What does n represent?*
>
> ***AP8:*** *First diagram where we have the number of wood, right.*

Half of the participants from the USA were able to construct quantities for the weight of the letter, the number of stamps, and postal delivery cost. However, due to their inability to see that functional relationship was based on the greater integer function they were not able to rationalize and see the relationship between the quantities. Even they had a less successful performance during the process of mathematizing of the first problem when compared to the other participants from USA, they were still able to clearly state the quantities in their native languages as follows:

> ***AP8:*** *As the number of stamps increase, I think this is how I wrote …. (thinks) ... so each one is required … the weight increased meaning the number of stamps increased as well. I mean they correlate with each other constantly. When the weight increases the number of stamps increases as well and the price of the stamps and the mailing letter accordingly. They correlate with one another because as the weight of the letter increases the price of the stamp increase by 0.21 cents.*
>
> ***I:*** *What did you come up with?*
>
> ***AP8:*** *I started with one point five which is basically thirty-one cents plus one point five times zero point 21 times … if ounces of fraction would be (stops talking) …. I am confused (thinking)….*

Two participants from the remaining four, however, had recognized the relationship within the problem right from the beginning by recalling the greatest integer function and were successful in the end.

> ***AP6:*** *I am not sure if my graph is right, but basically if we have an ounce there will be one stamp. And if we have one point one ounces like point nine, nine and nine ounces here, you will have two stamps...Maybe it shouldn’t be down here… (thinks) this is wrong; it shouldn’t be down here. And one, two …. So we have one ounce, we have one stamp. If we have two ounces, two stamps. The graphic should represent the relationship between the number of stamps and the weight of the letter. So I do not need to represent how much the stamps cost or the total cost of mailing letter (laughs, she notices her error in graphic formation). … So my graphic is actually wrong (draws again).*

4. DISCUSSION and CONCLUSION

Research results show that participants from Turkey had difficulties in terms of constructing quantities apart from the difficulties of not recognizing the functional relationship in word problems. In the first problem of the instrument, participants having difficulties in constructing quantities such as the number of stamps and stamp costs were not able to let go of the recursive thinking in the pattern problem and thus they were not able to recognize the functional relationship. The literature claims that (e.g. Zazkis & Liljedahl, 2002) recursive thinking has a very significant place in analyzing of change in a pattern; however, it has also been claimed that persistently thinking in recursive way prevents the development of algebraic thinking as well. Since the participants from Turkey were not able to go beyond the point of analyzing the pattern with recursive thinking they were not able to see the functional relationship within the pattern. As a result, they were not able to indicate this relationship neither verbally nor algebraically. When these results are taken into consideration in terms of these participants being middle school pre-service math teachers, the situation seems to be alarming. The development of quantitative reasoning and functional thinking during the process of the development of algebraic thinking is a part of the middle school curriculum. Students first construct quantities and then analyze relationships among quantities in quantitative reasoning (Thompson, 1989). For instance, according to Steele (2005) describing the patterns, expanding them and generalizing the relationships among quantities are the main components of generalization and are highly important for algebraic thinking in middle school grades. On the other hand, students' construction of quantities and relations among them can be possible only with the help of the mathematics teacher's guidance and questionings (Ellis, 2011). However, the literature asserts that the guidance and questioning, which help students, develop their quantitative reasoning and functional thinking skills will be possibly available only through teachers who have quantitative reasoning skills and functional thinking (e.g. Blanton & Kaput, 2011). Whether the participants in our study who have difficulties in quantitative reasoning and functional thinking can offer appropriate guidance and address suitable questions to the students, for the purpose of improving their way of thinking, still remains as an issue.

On the other hand, it was observed that most participants from USA did not face difficulties in terms of constructing the quantities in problems and in recognizing the relationships in quantities. It was observed that majority of these participants were clearly able to indicate the quantities and state relationship among them. However, it was evident that these participants faced difficulties in the process of mathematizing the problem and expressing the relationships algebraically even when they were able to express them verbally. Among the participants who were inclined to algebraically represent these verbally stated relationships without the use of a dependent variable concluded with algebraic statement instead of an algebraic equality. Similarly, Zazkis and Liljedahl (2002) concluded that the participants' explaining the relationships in the pattern verbally and having the ability to reach the symbolic rule were two discrete things. Zazkis and Liljedahl considered verbal statements as a product of algebraic thinking. They stated that students should be encouraged to think and express the problem cases verbally instead of being compelled to use symbols formally. Zazkis and Liljedahl emphasized that middle school pre-service mathematics teachers should be in study environments supporting their functional thinking during the process of pattern generalization. They also claim that providing a proper algebraic thinking for these candidates is much more important than using formal symbols.

On the other hand, participants from Turkey were more successful compared to the participants from the USA in transferring of informal sentences and statements to formal mathematical language. When the fact that these informal statements correlated with the number content was taken into consideration, American pre-service teachers' deficiency about the formal mathematical language knowledge and skills becomes evident. It is thought provoking that there was no American pre-service teacher being able to present the set representation fully. Therefore, American participants were not able to use the algebraic language as well. They were clearly able to express their mathematical thoughts verbally for problem cases whereas they did not have skills in symbolic representation in order to achieve the algebraic generalization of a pattern. It was evident that these participants faced difficulties in the process of mathematizing the problem and expressing the relationships algebraically even when they were able to express them verbally. According to Zazkis and Liljeahl (2002) who claimed that development of thinking skills and presentation (verbalization) skills precede symbolic representation skills, American students have the prerequisite skills for the development of symbolic representation skills since most of them have developed good thinking and verbalization skills. However, the difficulties participants from Turkey faced concerning the quantities and relationships among them in a pattern for middle school level and the symbols used when unpacking prove that these participant were not in the right path in terms of cognitive development. The training programs for pre-service mathematics teacher become crucial at this point. A good number of researchers together with Blanton and Kaput (2011) overemphasize in the literature as well that a great importance should be attached to a teacher's being able to think algebraically and functionally and to the ways of guidance for developing the students' ways of thinking algebraically. Middle school mathematics teacher training program applied in the university in Turkey includes mathematics courses like calculus, linear algebra and geometry as in mathematics major. However, this training program offers basic education courses such as classroom management conducted independently from the interpretation of field of mathematics as well as the mentioned higher mathematics courses. In this program, education and math field knowledge are only combined with the "special teaching methods" for two semesters. On the other hand, the training program of the university located in the USA does not have regular math courses without fundamental math courses like pre-calculus and calculus one variable. In this program the requirement three courses are algebraic thinking, statistical thinking, and geometric or special sense thinking. In these courses, pedagogical information is combining with middle school mathematics and aiming the development of mathematical thinking skills like quantitative reasoning and functional thinking.

It was concluded in this study that pre-service teachers, taking advanced level mathematics courses and education courses separately, have strong formal mathematics skills, but weak algebraic thinking skills. On the other hand, pre-service teachers, taking courses aiming the development of mathematical thinking skills and regular math courses have strong algebraic thinking skills, but weak formal mathematics skills. Middle school pre-service mathematics teachers are expected to be individuals having effective discourse skills in both native and mathematics language along with the ability to think quantitatively and functionally. Moreover, they should know how to support students' mathematics language skills and mathematical thinking skills as well. For that reason, these pre-service teachers deserve an appropriate training in accordance. Based on results of this study, such an appropriate training should include sufficient advanced level mathematics courses and some fundamental courses aiming to develop mathematical thinking skills. Apart from these courses existences, it was concluded that to give mathematics concepts in daily life situations and problems, not only in theoretical way is crucial. The reason of this conclusion is that even Turkish participants take a lot of mathematics courses including function concept, they were concluded not having the functional thinking skill.

Suggestions arising out of this study and suggestions to be made on future studies imply that this study should be taken into consideration comparatively with different and a large number of teacher training programs and that the quantitative and functional thinking skills, pedagogical approaches and the inter-relations of them should be studied in the training process. Such investigations will make an important contribution to the literature and the development of mathematics teacher training programs.

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Uzun Özet

Galileo matematiğin Tanrı'nın kainatı yazdığı dil olduğunu söylemiştir. Matematik diğer dillerde olduğu gibi kendine ait sözcük hazinesi ve gramatik yapısı olan bir dildir. Matematik dilinin diğer dillerden farklı olan yönü, anadilleri farklı bireyler tarafından farklı anadiller ile desteklenerek kullanılıyor olmasıdır. Matematiği evrensel dil yapan şey de bireylerin farklı anadil desteği ile matematiğe özgü sembolleri aynı biçimde gösterim ve aynı anlam ile kullanmalarıdır. Matematik dilinde ifade ve cümleler tamamen sembolik olabildikleri gibi sembollerin anadil kelimeleri ile birlikte, ardı ardına kullanıldıkları durumlar da vardır. Matematiksel ifadeler ya da cümleler sembol kullanılmaksızın, sembollerin anadildeki anlamları verilerek anadilde de ifade edilebileceğinden formal ve informal olmak üzere matematik dilinin iki tür kullanımı ortaya çıkmaktadır. İnformal dilden formal matematik diline dönüştürme, informal dilde verilmiş bir ifade ya da cümlenin sembolik yazılımı olabildiği gibi alan yazında bu dönüştürme genel olarak bir problem durumunun matematiksel modellere dönüştürülmesi olarak ele alınmaktadır.

Diğer yandan matematiğe dil perspektifinden bakıldığında ve öğretimde öğretmen rolü göz önüne alındığında, matematik dilini kazandırmaktan sorumlu olan matematik öğretmenlerinin matematik dil becerileri ve öğrencilerin matematik dil becerilerini geliştirme yaklaşımları önem kazanmaktadır. Alan yazında matematik öğretmenlerinin öğrencileri günlük yaşam dilinden matematik diline geçmeleri için desteklemelerinin gerekliliği vurgulanmaktadır. Bunun yanı sıra alan yazında matematik dilinin matematiksel kavramlar ile eşzamanlı olarak geliştirilmesinin gerekliliği vurgulanmaktadır. Matematik öğretmenlerinin matematiksel kavramların gelişimi sürecinde öğrencilerin matematiksel dil becerilerini geliştirmeleri için üstlenmeleri gereken sorumluluklar ise iki açıdan ele alınabilir. Bunlardan ilki matematik öğretmeninin sınıf ortamında yansıtacağı, kendisinin sahip olduğu matematiksel dil becerileridir. Matematik öğretmenleri yeteri kadar formal matematik dili kullanma becerisine sahip olmalı ve formal ve informal matematik dili arasında iki yönlü dönüştürme yapabilmelidirler. Matematik öğretmenlerinin sahip olmaları gereken yeterli matematik dil düzeylerinin yanı sıra öğrencilerin matematik diline maruz bırakılarak bu dili kazanamadıkları da alan yazında yer bulmuştur. Bunların sonucu olarak öğretmenlerin ikinci sorumluluğu olan öğrencilerinin matematik dili becerilerini geliştirmede sahip olmaları gereken pedagojik beceriler ortaya çıkmaktadır. Bu çalışmada ortaokul matematik öğretmen adaylarının informal dilden formal matematik diline dönüştürme becerilerinin incelenmesi amaçlanmıştır. Ayrıca ortaokul matematik öğretmeni yetiştirme programına göre bu becerilerin farklılaşma durumunun nasıl olduğu da bir araştırma sorusu olarak ele alındığından örneklem Amerika Birleşik Devletleri ve Türkiye'de bulunan birer devlet üniversitesindeki ortaokul matematik öğretmen adayları arasından seçilmiştir. Her iki üniversiteden de beşinci yarıyılında olan ve matematik derslerinin çoğunu almış olan ortaokul matematik öğretmen adayları arasından gönüllülük esasına bağlı kalınarak sekizer öğretmen adayı rastgele seçilmiştir. Nitel olarak desenlenen bu çalışmada veriler yazılı soru formu ve klinik görüşme yolu ile elde edilmiştir. Çalışmanın amacı doğrultusunda geliştirilen yazılı soru formu öğretmen adayı tarafından yanıtlandıktan sonra kendisi ile yanıtları konusunda klinik görüşme yapılmıştır. Araştırmadan elde edilen bulgular Amerikalı katılımcıların anadilde verilen matematiksel cümleleri formal matematik diline dönüştürme (unpack) konusunda oldukça zayıf, Türkiye'den katılımcıların ise günlük yaşam problemlerini matematikleştirme sürecinde zayıf olduklarını göstermiştir. Dönüştürme konusunda matematik dilini Amerikalı katılımcılara göre daha etkili kullanan, cebirsel temsil oluşturma ve cebirsel manipülasyonlarda daha başarılı oldukları görülen Türkiye'den katılımcıların problemlerde nicelikleri oluşturma, nicelikler arası ilişkileri kurma ve bunları uygun biçimde matematiksel temsiller ile gösterebilme konusunda güçlükler yaşadıkları görülmüştür. Anadillerinde verilen ifade ve cümleleri formal matematik diline dönüştürme konusunda güçlükler yaşayan Amerikalı katılımcılar ise matematiksel düşüncelerini sözlü temsil etme konusunda ve problemleri matematikleştirme sürecinde daha başarılı olduklarını göstermişlerdir.

Problemlerde fonksiyonel ilişkiyi tanıyamama güçlüğü bir yana nicelikleri oluşturma konusunda güçlükler yaşayan Türkiye'den katılımcıların ortaokul matematik öğretmen adayları olmaları göz önüne alınarak düşünüldüğünde, beşinci yarıyıllarında olan ve niceliksel muhakeme ve fonksiyonel düşünmede güçlükleri olan bu ortaokul matematik öğretmen adaylarının öğrencilerde bu düşünmelerin gelişebilmesi için uygun yönlendirme ve sorgulamalar yapabilme olasılıkları tartışma konusudur. Cebirsel düşünmenin gelişimi sürecinde niceliksel muhakemenin ve fonksiyonel düşünmenin gelişimi ortaokul yıllarına denk gelir. Alan yazında öğrencilerin bir problem durumundaki nicelikleri oluşturmalarının ve ilişkilendirmelerinin öğretmenin yaptığı yönlendirmeler ve sorgulamalarla mümkün olduğu vurgulanmaktadır. Ayrıca öğrencilere onların niceliksel muhakeme ve fonksiyonel düşünme becerilerini geliştirecek yönlendirme ve sorgulamalar, niceliksel muhakemeye sahip, fonksiyonel düşünebilen öğretmenlerle mümkün olacağı savunulmaktadır. Ölçme aracında informal olarak verilen cümle ve ifadeleri formal matematik diline aktarma konusunda ise Türkiye'den katılımcılar başarı gösterirken, Amerika'dan öğretmen adayları zayıf performans sergilemişlerdir. Verilen informal cümlelerin sayı kavramı ile ilişkili olduğu göz önüne alındığında ise Amerika'dan öğretmen adaylarının formal matematik dili bilgi ve becerilerinin zayıflığı ortaya çıkmaktadır. Örneğin küme temsilini tam olarak verebilen öğretmen adayının bulunmaması düşündürücüdür. Ayrıca bu öğretmen adaylarının formal matematik dili becerilerindeki zayıflık cebir dilini kullanamama boyutundadır. Problem durumları ile ilgili matematiksel düşüncelerini sözlü olarak açıkça ortaya koyabildiklerini ancak örüntünün cebirsel genellemesine ulaşamama boyutunda sembolik temsil becerisine sahip olmadıklarını gösteren bu öğretmen adaylarından beklenen ise öğrencilerinde örüntülerin cebirsel kuralına ulaşma becerilerini desteklemek olacaktır. Alan yazında ortaya konulan, düşünme becerilerinin ve sözlü temsil becerisinin gelişiminin sembolik temsil becerisinden önce geldiği sonucu göz önüne alındığında Amerika'dan öğretmen adaylarının sembolik temsil becerilerinin geliştirilmesi için önkoşul becerilere sahip oldukları ancak bu gelişimlerinin eksik kaldığı düşünülmektedir. Bu sonuçlar karşısında önem kazanan şeylerden birisi matematik öğretmen adaylarının eğitim programlarıdır. Alan yazında ortaokul matematik öğretim programlarında, öğretmen adaylarının cebirsel ve fonksiyonel düşünebilmelerine ve öğrencilerinin cebirsel düşünme yollarını geliştirme konusundaki yönlendirme biçimlerine önem verilmesine vurgu yapılmaktadır. Bu araştırmanın yürütüldüğü iki üniversitedeki ortaokul matematik öğretmeni eğitim programları göz önüne alındığında, Amerika'daki üniversitede uygulanmakta olan öğretim programında alana özgü öğretim yöntemlerinin verildiği derslerle birlikte pedagojik bilgi ile ortaokul matematiğinin birleştirildiği ve niceliksel muhakeme, fonksiyonel düşünme gibi matematiksel düşünme becerilerinin gelişimini hedefleyen dersler dikkat çekmektedir. Türkiye'deki üniversitede uygulanan ortaokul matematik öğretmen eğitim programında ise eğitim ve matematik alan bilgisi yalnızca iki dönemlik "özel öğretim yöntemleri" dersi ile birleştirilmektedir. Türkiye'de uygulanan eğitim programında öğretmen adaylarının cebirsel düşünme becerilerini destekleyen derslere olan ihtiyaç ortaya çıkarken, Amerikalı öğretmen adaylarından elde edilen sonuçlar ise kendilerine uygulanan eğitim programında göze çarpan matematik alan derslerinin azlığını ortaya koymaktadır. Sonuç olarak ortaokul matematik öğretmen adayları eğitim programlarında yeteri kadar formal matematik dil bilgisi verecek alan dersinin ve cebirsel düşünme becerisini destekleyen derslerin gerekliliği ortaya çıkmaktadır.

Bu çalışmadan doğan ve gelecek araştırmalara yapılabilecek öneriler araştırmanın farklı ve daha çok sayıda öğretmen eğitim programları ile karşılaştırmalı yürütülmesi ve ortaokul matematik öğretmen adaylarının niceliksel ve fonksiyonel düşünme becerilerinin, pedagojik yaklaşımlarının ve bunlar arasındaki ilişkilerin öğretmen eğitim sürecinde incelenmesi yönündedir. Bu inceleme alan yazına, matematik öğretmen eğitim programlarının geliştirilmesine yönelik katkı sağlayacaktır.