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THE STUDY OF DEPENDENCE OF THE RADIAL PARTS OF $0^- \rightarrow 0^+$ FIRST FORBIDDEN β-DECAY MATRIX ELEMENTS ON THE PARAMETERS IN WOODS-SAXON POTENTIAL

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Abstract-In this paper, $0^- \rightarrow 0^+$ first forbidden β-decay matrix elements have been calculated. For $^{206\text{-}214}Pb \rightarrow ^{206\text{-}214}Bi$ transitions relativistic $M^{\pm}(\rho_A, \lambda=0)$ and nonrelativistic $M^{\pm}(j_A, \kappa=1, \lambda=0)$ matrix elements have been calculated. In this calculations, the eigenfunctions and eigenvalues of Schrodinger equation solved by the more realistic Woods-Saxon potential have been used. The dependence of the radial parts in the matrix elements on the parameters (r_0: the radii of nucleus, η: isovector parameter, r_c: Coulomb radius and χ diffusion parameter) of Woods-Saxon potentialhas been investigated.

Keywords-Beta decay matrix element.

1. INTRODUCTION

First forbidden beta decay process have been studied "for single-particle configurations in the region of ^{208}Pb" by Bohr [1]. First forbidden resonance (FFR) has been studied to confirm that the spin-isospin vibration mode of (p, n) reactions exists [2-4]. Descriptions of the Gamow-Teller Resonance (GTR) and FFR have been reported for deformed nuclei [5,6]. Civitarese & et al. studied the Collective Effects of Charge-Exchange Vibrational modes on $0^- \rightarrow 0^+$ and $2^- \rightarrow 0^+$ first-forbidden β-decay [3,7]. As known, the theoretical description of nuclear double beta decay processes is one of the open questions in the field of nuclear structure theories. The use of intermediate virtual excitations other than the allowed ones has been advocated by some authors [7-9] in spite of the fact [10] that the leptonic wave functions include terms which are proportional to the product of the electron or neutrino velocity with the nuclear radius.

Many researchers preferred "the use of virtual excitations" (which include additional terms) instead of "allowed ones". Generally, the calculations for $0^- \rightarrow 0^+$ First Forbidden β-Decay have been made in the base of harmonic oscillator [3-5].

In this study, $0^- \rightarrow 0^+$ first forbidden β-decay transitions for the nucleus $^{206\text{-}214}Pb \rightarrow ^{206\text{-}214}Bi$ have been studied. The calculations have been made in the Woods-

Saxon potential base. $M^{\pm}(\rho_A,\lambda=0)$ matrix element has been calculated analytically. Dependency of the Radial parts of the matrix element on the Woods-Saxon potential has been examined.

2. FORMALISM

In the "ξ approximation" the decay rates for $0^- \to 0^+$ transition can be written in the following form [1].

$$f_0 t(B(\lambda\pi=0^-)=Dg_v^2/4\pi\,, \tag{1}$$

where,

$$B(\lambda\pi=0-)=\frac{1}{2I_i+1}\left|\left\langle I_f\left\|\pm M(\rho_A,\lambda=0)-i\frac{m_e c}{\hbar}\xi.\mathrm{M}(j_A,\kappa=1,\lambda=0)\right\|I_i\right\rangle\right|^2 .$$

Relativistic matrix element is:

$$M(\rho_A,\lambda=0)=(4\pi)^{-1/2}\frac{g_A}{c}\sum_k t_-(k)(\sigma(k)\cdot v_k)\,, \tag{2}$$

and nonrelativistic matrix element is:

$$M(j_A,\kappa=1,\lambda=0)=g_A\sum_k t_-(k)\,r_k(Y_1(\hat{r}_k)\sigma(k))_0\,, \tag{3}$$

$\xi \approx 1.2Z.A^{-1/3}$, D=6250sec.

In these notes the upper and lower signs refer to β^-, β^+ decays, respectively. g_A and g_v refer to axial and vector coupling constant respectively. It can be shown that reduced matrix elements of the operators given in the formulas (2), (3) are proportional to the expressions given below [11-13].

$$\left\langle J,p\left\|r_k\left(Y_1(\hat{r}_k)\sigma(k)\right)_0\right\|J,n\right\rangle=(-1)^{\ell_n+J-1/2}\cdot\sqrt{\frac{(2J+1)(2\ell_n+1).6}{4\pi}}\cdot\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & 1\\ \ell_n & \ell_p & J\end{Bmatrix}\left\langle \ell_n 010\middle|\ell_p 0\right\rangle R_{np}\,,$$

$$J_n=J_p=J,$$

$$R_{np}=\int_0^{\infty}U^*_{l_p}(r)\cdot r\cdot U_{l_n}(r)dr\,,$$

$$\left\langle J_1(p)\right\|\left(\vec{\sigma}\vec{\nabla}\right)\left\|J_2(n)\right\rangle=\sqrt{2J_2+1}\,\delta_{J_1J_2}\delta_{m'm}(-1)^{J_2+\ell_1-1/2}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & 1\\ \ell_2 & \ell_1 & J_2\end{Bmatrix}$$
$$\sqrt{6}\left(\sqrt{\ell_2+1}\,A_{\ell_1\ell_2}\delta_{\ell_1,\ell_2+1}-\sqrt{\ell_2}\,B_{\ell_1\ell_2}\delta_{\ell_1,\ell_2-1}\right),$$

$$A_{\ell_p\ell_n}=\int_0^\infty\psi^*_{n_pl_p}(r)\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\psi_{n_nl_n}(r)r^2dr\,,$$

$$B_{\ell_p\ell_n}=\int_0^\infty\psi^*_{n_pl_p}(r)\left(\frac{\partial}{\partial r}-\frac{l_n+1}{r}\right)\psi_{n_nl_n}(r)r^2dr\,.$$

Where, $\psi_{n\,l}$ is the radial part of wave function, n is the main quantum number and l is the orbital angular momentum quantum number.

2. RESULTS AND DISCUSSIONS

In our calculations, the eigenfunctions and eigenvalues of Schrodinger equation solved by Woods-Saxon potential have been used. The matrix elements which are calculated with the formulas (2) and (3) depend on the Woods-Saxon potential parameter [13]. The dependency of the radial section on the potential parameter has been examined as that dependency occurred with radial part.

The reduced matrix elements determining the transition probability from the ground state (0^+) of ^{210}Pb isotope to the first 0^- state of ^{210}Bi isotope have been calculated. Since the first 0^- state of ^{210}Bi isotope is not collective the structure of this state is assumed to be $(3s^h_{1/2}\to 3p^p_{1/2})$ or $(1g^h_{9/2}\to 1h^p_{9/2})$. Therefore, the reduced matrix elements of the operators given in the formulas (2) and (3) for the transitions between the 0^+ ground state of ^{210}Pb and 0^- state of ^{210}Bi have been calculated and the dependence of their radial parts on Woods-Saxon potential parameter have been investigated. The calculation results are shown in Figs. 1-3.

The dependence of the radial integrals in relativistic and nonrelativistic matrix elements on the radii of nucleus (r_0) which is one of the parameters in the average field potential is shown in Figs. 1a and 1b. It is clearly seen that there is no change in the radial parts of these matrix elements even though the radii of nucleus varies in the range of (1.24-1.34)fm.

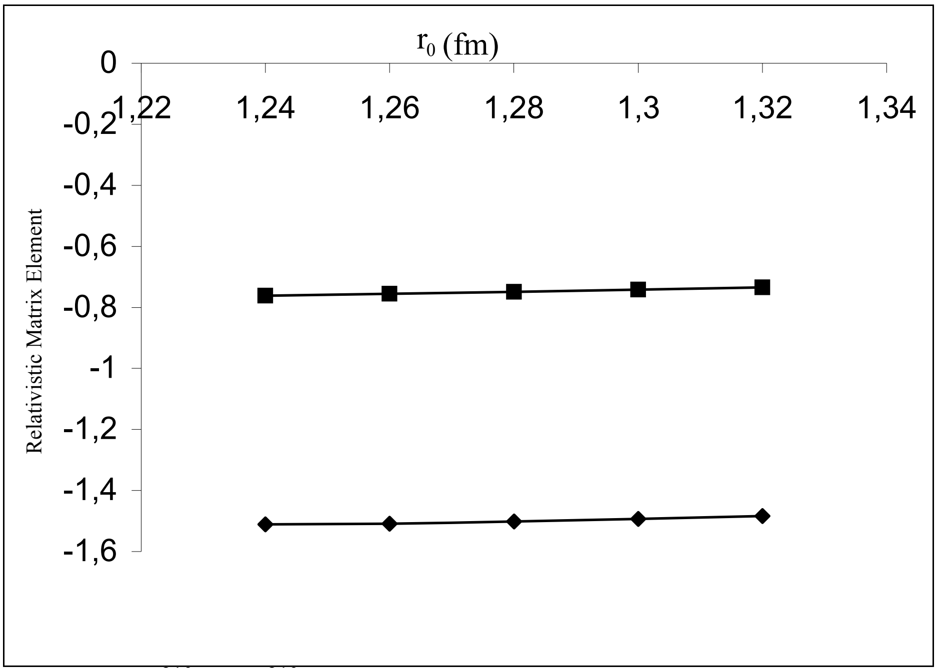

Fig. 1a: For $^{210}Pb \rightarrow {}^{210}Bi$ transition, the dependence of the radial part of $0^{+} \rightarrow 0^{-}$first forbidden beta decay relativistic matrix elements on the radii of nucleus

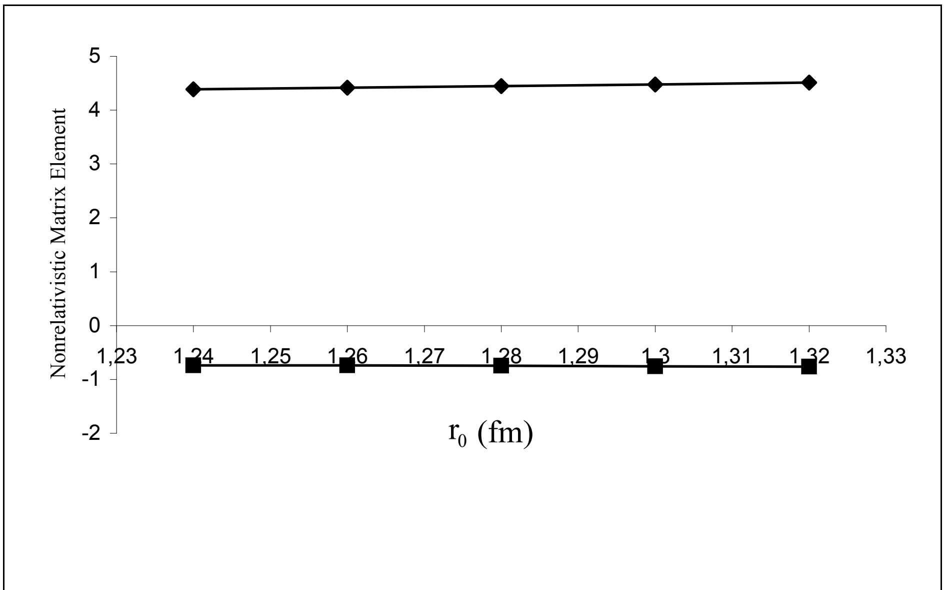

Fig. 1b: For $^{210}Pb \rightarrow {}^{210}Bi$ transition, the dependence of the radial part of $0^{+} \rightarrow 0^{-}$first forbidden beta decay nonrelativistic matrix elements on the radii of nucleus

However, it is seen that the signed statements at first is related to the isovector parameter in the Woods-Saxon Potential at the fig. 2a and fig. 2b.

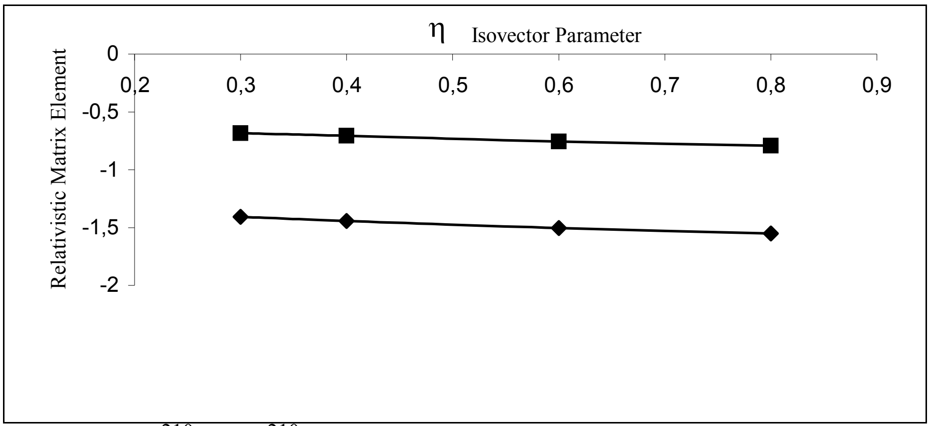

Fig. 2a: For $^{210}Pb \to {}^{210}Bi$ transition, the dependence of the radial part of $0^+ \to 0^-$ first forbidden beta decay relativistic matrix elements on η isovector parameter

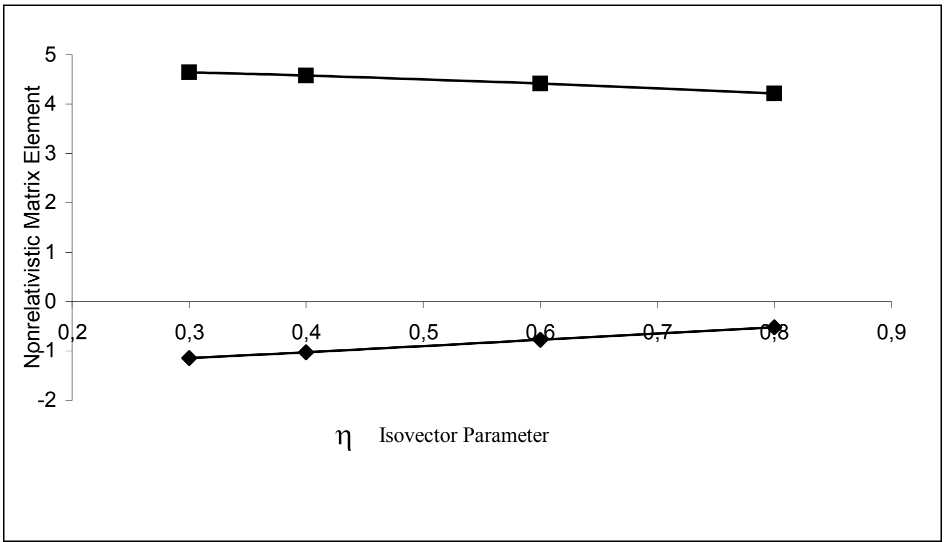

Fig. 2b: For $^{210}Pb \to {}^{210}Bi$ transition, the dependence of the radial part of $0^+ \to 0^-$ first forbidden beta decay nonrelativistic matrix elements on η isovector parameter

It is seen that the dependence on the Coulomb radius shown in figs. 3a and 3b is similar to the dependence on the isovector parameter.

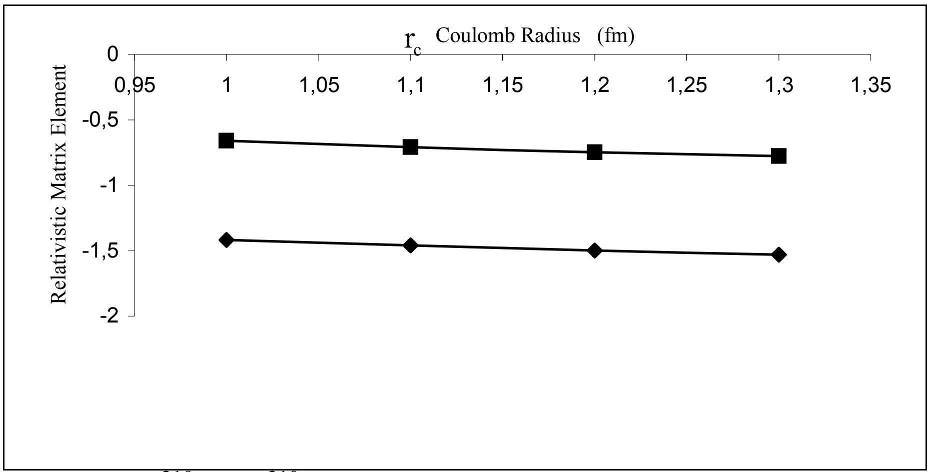

Fig. 3a: For $^{210}Pb \rightarrow ^{210}Bi$ transition, the dependence of the radial part of $0^+ \rightarrow 0^-$ first forbidden beta decay relativistic matrix elements on Coulomb radius

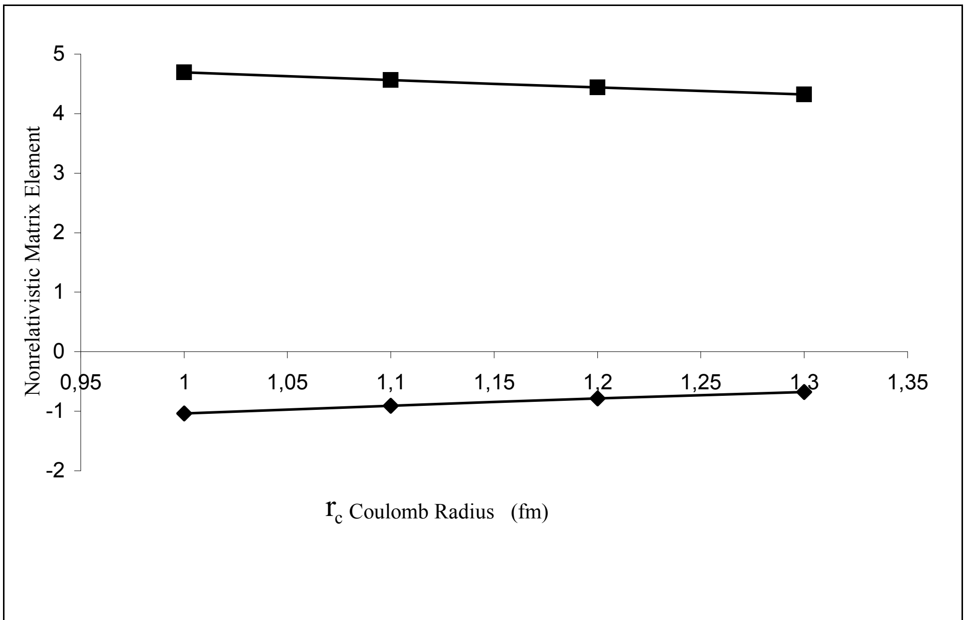

Fig. 3b: For $^{210}Pb \rightarrow ^{210}Bi$ transition, the dependence of the radial part of $0^+ \rightarrow 0^-$ first forbidden beta decay nonrelativistic matrix elements on Coulomb radius

In fig. 4a and fig. 4b, the dependence of these radial matrix elements on the diffusion parameter has been shown. The relationship between the radial matrix elements and diffusion parameter is completely different from other relations.

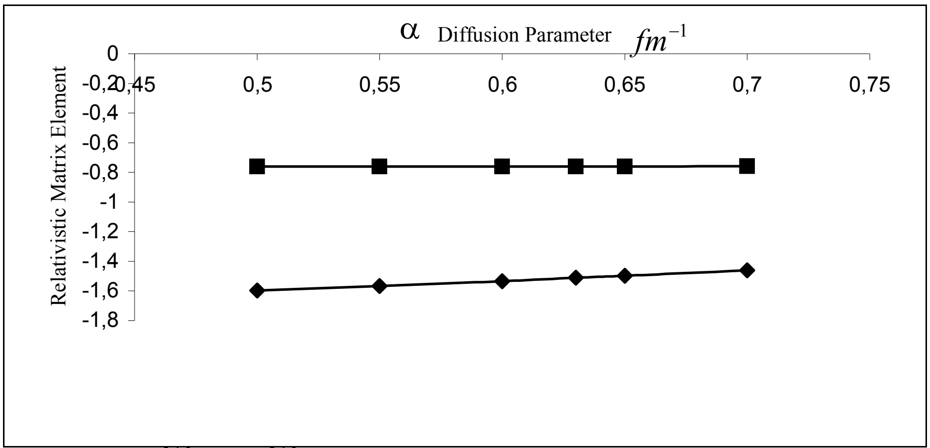

Fig. 4a: For ^{210}Pb$\rightarrow$^{210}Bi transition, the dependence of the radial part of $0^+ \rightarrow 0^-$first forbidden beta decay relativistic matrix elements on the diffusion parameter

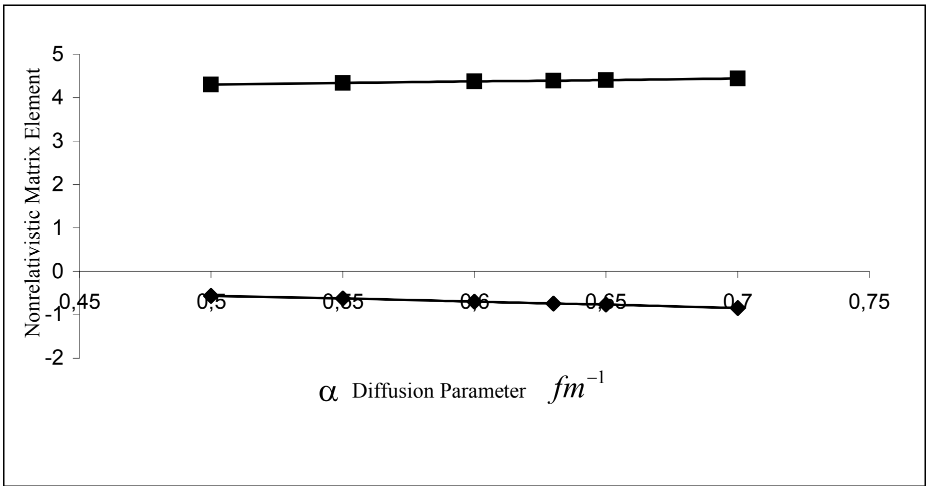

Fig. 4b: For ^{210}Pb$\rightarrow$^{210}Bi transition, the dependence of the radial part of $0^+ \rightarrow 0^-$first forbidden beta decay nonrelativistic matrix elements on the diffusion parameter

Based on our calculations, below statements could be said.

1. Dependence of the matrix elements on the Woods-Saxon parameters is weak in the study region.
2. The values of the first matrix element, which we calculated directly, are very different from the values calculated in Ref.[3].

The future study will contain the calculations of the *log ft* values for the $0^- \to 0^+$ transitions with the addition of spin orbit effect to the relativistic matrix element.

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