A Study on Estimation of Wind Speed Distribution by Using the Modified Weibull Distribution

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Abstract- The knowledge of the probability density function of wind speed is key information for the determination of wind energy potential of the specified region. In the literature, the Weibull distribution is a widely-used and accepted distribution to express the probability density function of wind speed data. However, the Weibull distribution does not exhibit a good fitting for all wind speed data measured at different geographical locations throughout the world. Thus, in this study, it is proposed that a better fitting of wind speed data and a better estimating wind power density are possible with the new modified Weibull distribution (MWD). We also compare the performance of the MWD relative to the Weibull distribution by using wind speed data measured in different regions of Turkey. The results state that the MWD shows good fitting for the most of the considered wind speed data cases. Thus, the MWD can be an alternative tool for the assessment of wind energy potential.

Keywords- Wind speed data, The Weibull distribution, The Modified Weibull distribution, Least square estimator, Wind power.

1. INTRODUCTION

Energy is one of the serious issues associated with future of the world. Because of their benefits, renewable energy sources attract great attention in the world day by day. Wind energy is now one of the most cost-effective and efficient sources among renewable energy sources. Turkey, as a developing country, has great wind energy potential. In this context, wind energy as natural resources has a critical importance for Turkey [1].

It is well-known that wind energy potential can be estimated by using the distribution of the wind speed. Thus, finding suitable wind speed distribution is one of the most important steps for the accurate estimation of wind energy potential of a specific region. In wind energy applications, two parameters Weibull distribution (WD) is the widely used and accepted distribution for estimating wind energy potential thanks to WD's computable and flexible mathematical form [2, 3].

The probability density function (pdf) of the WD is given as follows:

$$f(x)=\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}, \quad x>0, \alpha>0, \beta>0 \quad (1)$$

where β and α are shape and scale parameters, respectively.

The corresponding cumulative distribution function (cdf) is:

$$F(x)=1-e^{-\left(\frac{x}{\alpha}\right)^{\beta}}, \quad x>0. \quad (2)$$

Also, different patterns for pdf of the WD are presented in Figure 1.

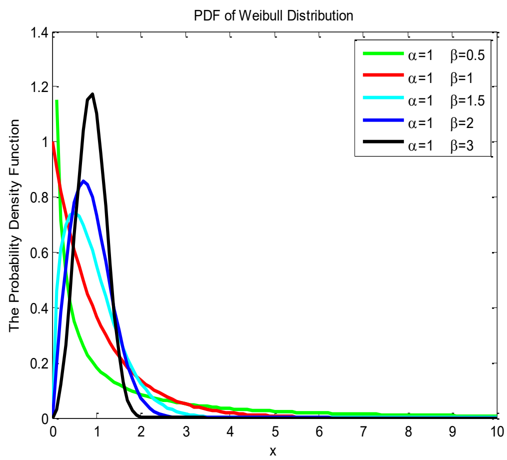

Figure 1. Different patterns for pdf of the WD.

For wind speed and power analyses, the WD is examined substantially. However, the WD does not always give accurate results for all wind speed types. This issue has been demonstrated in studies. For instance, the WD is not

well describe to wind anisotropy and wind statistics. Also this situation leads to a systematic under estimation of wind power, up to 12% [24]. In this context, different flexible distributions have been used to model wind speed data in literature [4-17]. For example,[4-7] model wind speed with two mixture Weibull distribution. [8, 9] consider maximum entropy principle for estimating distribution wind speed. Similarly, the distributions based on maximum entropy and minimum cross entropy principles are proposed by [10, 11].
[12] compares the Gamma, log-normal, Rayleigh and Weibull distributions in terms of modeling wind speed data. Also, for wind speed, [13, 14] use various statistical distributions, such as Erlang, Inverse Normal and Gumbel-Maximum distributions. Besides these distributions, mixture distributions are tested by [25] for wind speed distribution. A good number of novel distribution models are introduced for modeling wind speed in the literature [15-19].

In conclusion, the mentioned studies highlight that the WD shows a poor fitting for wind speed data when compared with the distribution with more parameters.

In this context, various extensions or modified forms of the WD have been studied to increase flexibility of the WD[20-23]. One of them is the modified Weibull distribution (MWD) introduced by Lai et al. (2003) [20].The MWD is more flexible than the WD due to the number of parameters. Also the MWD nests the WD as a special cases.

Thus, in this paper, the MWD is proposed to model wind speed data as a good alternative for the WD. For this purpose, availability of the MWD is studied for assessment of wind energy potential. Moreover, the MWD is compared with the WD for wind speed data measured in the different regions of Turkey.

The remainder of this paper is organized as follows: In Section 2, the MWD is introduced for modeling wind speed data. Section 3 presents the results of calculation and analysis concerning the MWD and the WD. Finally, the study is concluded with some outcomes in Section 4.

2. THE MODIFIED WEIBULL DISTRIBUTION FOR WIND SPEED DISTRIBUTION

The MWD proposed by Lai et al. (2003) [20] is introduced in this section. The cdf of the MWD is given as:

$$F(x) = 1 - e^{-\alpha x^{\beta} e^{\lambda x}}, \quad x > 0,\ \alpha > 0,\ \beta > 0,\ \lambda \geq 0. \qquad (3)$$

where λ and β are shape parameters, α is the scale parameter. Corresponding pdf for the MWD is:

$$f(x) = \alpha(\beta + \lambda x)x^{\beta-1} e^{\lambda x} e^{(-\alpha x^{\beta} e^{\lambda x})} \qquad (4)$$

The MWD includes the WD as a special case when λ is equal to zero. Thus, the MWD is a more flexible family than the WD.
Graphs of pdf for the MWD are shown for different parameter values in Figure 2. Also, it is clear that the graph of the MWD is same with corresponding graph of the WD at first pattern in Figure 2. It is concluded from Figure 2 that the MWD having the ability of various fitting can be adapted for modelling actual wind data cases.

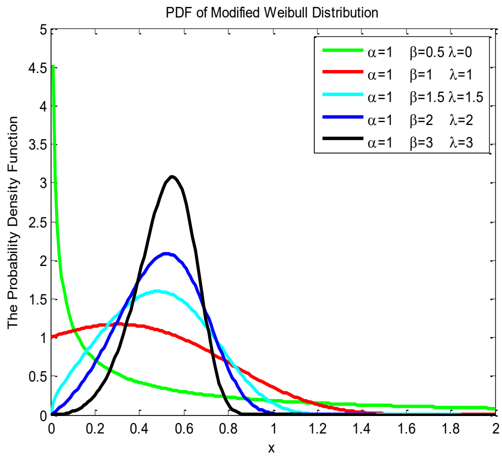

Figure 2. Different patterns for PDF of the MWD.

In order to find the unknown parameters of the MWD and the WD, different parameter estimation methods can be used [19,21,22]. In this study, we consider the least square method (LSM) to estimate the unknown parameters of the MWD due to its easy usage and computation.

As known, the linearization of a cdf is required for the use of the LSM. The linearization process for the MWD model is implemented as follows:

The cdf of the MWD can be rewritten by using the Equation (3) as a linear model:

$$\ln[-\ln(1 - f(x))] = \ln\alpha + \beta \ln(x) + \lambda x \qquad (5)$$

In this equation, we replaced $\ln[-\ln(1 - f(x))]$ with y, $\ln\alpha$ with β_0, β with β_1, $\ln(x)$ with x_1, λ with β_2 and x with x_2 thus, the Equation (5) is rewritten as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \qquad (6)$$

By using this linear regression model given in the Equation (6), LSM can be easily employed to estimate the parameters ofthe MWD without any numeric computational difficulty. For interpretation and comparison of the obtained results, the parameters of the WD are estimated by using LSM.

3. ANALYSIS AND RESULTS

To compare the WD and MWD, monthly and yearly wind speed data obtained from two different regions in Turkey is used. The hourly wind speed data measured at 10 m above ground level in Bandırma and Kireçburnu regions of Turkey is used for all comparions. Descriptive statistics regarding these regionsare given in Table 1 and Table 2, respectively. Also, box-plots of considered wind data are respectively illustrated in Figure 3 and Figure4.

Table 1. Descriptive statistics for monthly and yearly wind speed data (m/s) for Bandırma, Turkey (1997).

	Mean	Variance	Skewness	Kurtosis	n
Jan.	3.885	10.931	1.033	3.529	736
Feb.	3.947	10.738	0.960	3.096	671
Mar.	4.497	11.327	0.618	2.571	742
Apr.	3.471	5.573	0.843	3.348	715
May	3.776	5.561	0.613	2.906	739
Jun.	3.164	4.747	0.641	2.799	699
Jul.	3.268	4.995	0.501	2.478	706
Aug.	4.046	7.335	0.532	2.511	714
Sept.	5.018	8.561	-0.060	1.934	688
Oct.	4.764	13.349	0.554	2.037	740
Nov.	3.006	5.540	0.943	3.276	686
Dec.	3.901	9.698	1.197	3.770	741
Year	3.901	8.574	0.856	3.183	8577

Table 2. Descriptive statistics for monthly and yearly wind speed data (m/s) for Kirecburnu, Turkey (1996).

	Mean	Variance	Skewness	Kurtosis	n
Jan.	2.936	5.427	0.774	2.814	691
Feb.	2.076	3.820	1.310	4.016	565
Mar.	1.790	1.818	0.805	3.073	633
Apr.	1.798	1.392	0.435	2.586	645
May	1.694	1.308	0.591	3.142	641
Jun.	1.672	1.078	0.157	1.963	635
Jul.	2.536	1.860	-0.065	2.201	702
Aug.	1.847	1.499	0.257	2.010	656
Sept.	1.947	2.136	0.422	2.077	632
Oct.	2.416	3.717	0.711	3.003	615
Nov.	1.720	1.338	0.549	2.928	620
Dec.	1.713	2.003	0.887	2.873	619
Year	2.022	2.438	1.067	4.537	7654

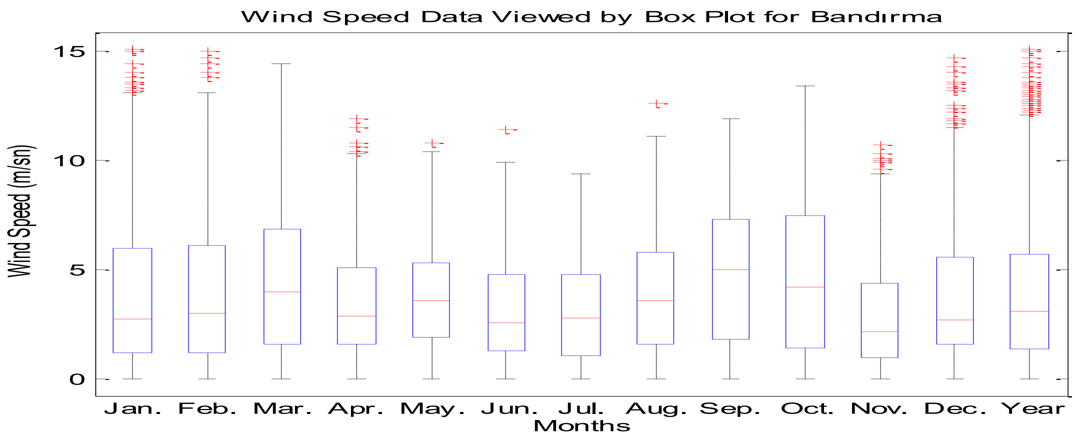

Figure 3. Box-plots for monthly and yearly wind speed data for Bandırma (1997)

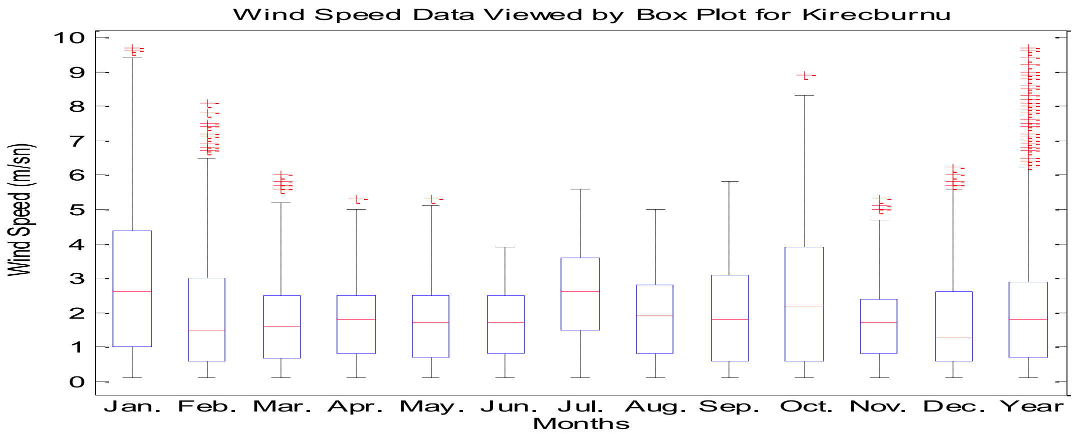

Figure 4. Box-plots for monthly and yearly wind speed data for Kireçburnu (1996)

The variation for different time periods and regions can be seen from Table 1-2 and Figures 1-2, clearly. In other words, Bandırma's data has higher mean and standard deviation values than Kireçburnu for all time periods. Considering both skewness and kurtosis coefficients, Bandırma has the highest values relative to Kireçburnu for the most considered time periods. Thus, it can be concluded that these regions have different statistical characteristics of wind speed.

The coefficient of determination (R^2), the root mean square error (RMSE) and the Kolmogorov-Smirnov (K-S) are used to evaluate the suitability of the MWD. Detailed information about model selection criteria is included in [11, 16, 17]. Also, power density error (PDE) is used to evaluate the capability of distribution in estimating wind power.

The formulations of criteria and PDE for model evaluation are given in Table 3.

Table 3. The formulas of criteria for model evaluation

Criteria	Formulas
R^2	$1-\sum_{i=1}^{N}(y_i-x_i)^2 \Big/ \sum_{i=1}^{N}(y_i-z_i)^2$
RMSE	$\left(\sum_{i=1}^{N}(y_i-x_i)^2 \Big/ N\right)^{1/2}$
K-S	$\max_{1\le i\le N}\left(F(v_i)-(i-1)/N, i/N-F(v_i)\right)$
PDE	$\left\lvert\dfrac{P_{REF}-P_D}{P_{REF}}\right\rvert \times 100$

The estimated values of parameters and the results of criteria, K-S, R^2 and RMSE, corresponding to the WD and the MWD are presented for Bandırma and Kirecburnu in Table 4 and Table 5, respectively. Also, the estimated values of parameters for the WD and the MWD are demonstrated in Figures 5-6 and Figures 7-8.

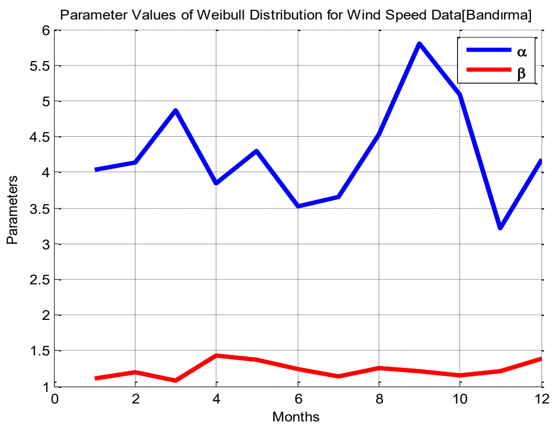

Figure 5. The estimated values of parameters of the

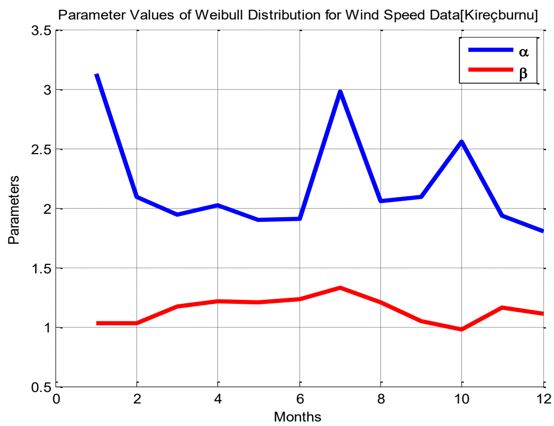

Figure 6. The estimated values of parameters of the

WD for monthly wind speed data of Bandırma.

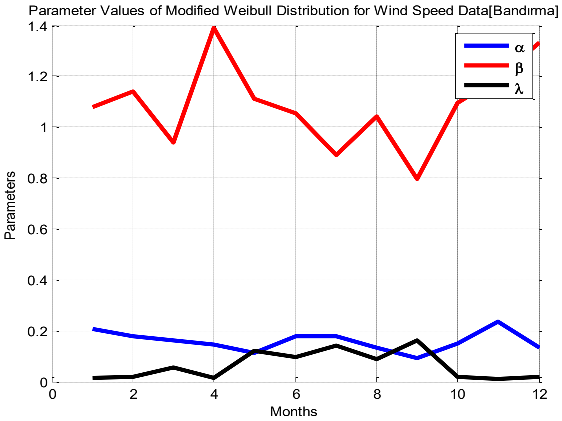

Figure 7. The estimated values of parameters of the MWD for monthly wind speed data of Kireçburnu.

WD for monthly wind speed data of Kireçburnu.

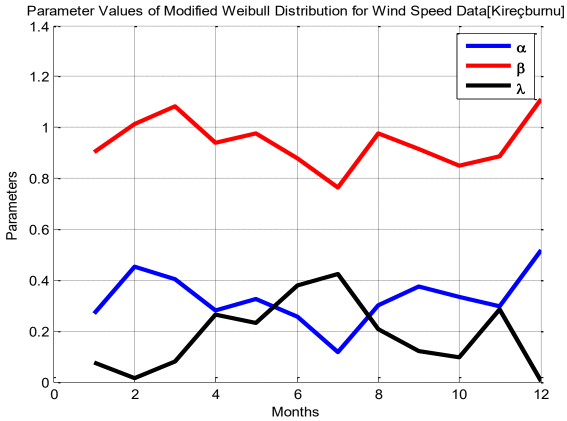

Figure 8. The estimated values of parameters of the MWD for monthly wind speed data of Kireçburnu.

Table 4. Estimations of parameters of the WD and the MWD and the results of criteria for Bandırma.

	α	β	λ	R^2	RMSE	KS
			January			
WD	4.027	1.115	----	0.914	0.002	0.046
MWD	0.206	1.077	0.015	0.914	0.002	0.053
			February			
WD	4.132	1.196	----	0.885	0.003	0.059
MWD	0.178	1.141	0.020	0.879	0.003	0.062
			March			
WD	4.872	1.080	----	0.882	0.002	0.051
MWD	0.162	0.940	0.055	0.914	0.002	0.033
			April			
WD	3.846	1.426	----	0.970	0.001	0.022
MWD	0.143	1.389	0.015	0.970	0.001	0.025
			May			
WD	4.299	1.378	----	0.905	0.002	0.050
MWD	0.113	1.111	0.118	0.948	0.001	0.031
			June			
WD	3.523	1.236	----	0.925	0.002	0.053
MWD	0.179	1.055	0.096	0.948	0.002	0.024
			July			
WD	3.660	1.137	----	0.862	0.002	0.061
MWD	0.176	0.889	0.139	0.964	0.001	0.027
			August			
WD	4.530	1.249		0.887	0.002	0.048
MWD	0.132	1.042	0.087	0.934	0.001	0.018
			September			
WD	5.805	1.211	----	0.267	0.004	0.126
MWD	0.090	0.795	0.161	0.674	0.002	0.043
			October			
WD	5.079	1.150	----	0.735	0.003	0.053
MWD	0.150	1.096	0.018	0.726	0.003	0.051
			November			
WD	3.219	1.217	----	0.969	0.001	0.032
MWD	0.236	1.195	0.011	0.969	0.001	0.033
			December			
WD	4.175	1.392	----	0.874	0.003	0.089
MWD	0.134	1.331	0.020	0.856	0.003	0.095
			Whole			
WD	4.224	1.225	----	0.977	0.000	0.019
MWD	0.165	1.173	0.021	0.981	0.000	0.022

Table 5. Estimations of parameters of the WD and the MWD and the results of criteria for Kireçburnu.

	α	β	λ	R^2	RMSE	KS
			January			
WD	3.129	1.027	----	0.949	0.002	0.048
MWD	0.268	0.904	0.073	0.972	0.001	0.043
			February			
WD	2.092	1.034	----	0.970	0.002	0.060
MWD	0.453	1.013	0.015	0.970	0.002	0.059
			March			
WD	1.940	1.173	----	0.963	0.002	0.047
MWD	0.405	1.084	0.079	0.981	0.001	0.049
			April			
WD	2.023	1.214	----	0.839	0.004	0.065
MWD	0.281	0.940	0.264	0.965	0.002	0.048
			May			
WD	1.895	1.207	----	0.867	0.004	0.091
MWD	0.324	0.977	0.230	0.957	0.002	0.057
			June			
WD	1.904	1.233	----	0.678	0.005	0.103
MWD	0.256	0.877	0.377	0.947	0.001	0.053
			July			
WD	2.976	1.328	----	0.362	0.006	0.115
MWD	0.116	0.765	0.423	0.917	0.002	0.040
			August			
WD	2.060	1.202	----	0.823	0.004	0.076
MWD	0.301	0.976	0.208	0.899	0.002	0.053
			September			
WD	2.093	1.052	----	0.862	0.004	0.070
MWD	0.373	0.916	0.119	0.893	0.003	0.067
			October			
WD	2.554	0.978	----	0.873	0.004	0.075
MWD	0.331	0.848	0.095	0.911	0.003	0.063
			November			
WD	1.934	1.165	----	0.842	0.005	0.073
MWD	0.298	0.888	0.283	0.962	0.002	0.061
			December			
WD	1.802	1.113	----	0.987	0.001	0.050
MWD	0.516	1.111	0.002	0.987	0.001	0.050
			Whole			
WD	2.186	1.135	----	0.961	0.000	0.050
MWD	0.355	1.029	0.087	0.985	0.000	0.049

It is observed from Table 4 that the MWD shows a better performance relative to the WD for the most of the considered months and the whole year in terms of all criteria. Also, it is seen from Table 5 that the MWD providesa better fitting than the WD for almost all months and whole year due to the smallest RMSE and KS and the highest R^2 values.

It is known that the mean wind power density based on distributional model (P_D) is calculated from the following formula:

$$P_D = \frac{1}{2}\rho A \int_0^{\infty} v^3 f(v) dv \tag{7}$$

where v is wind speed, ρ is air density (kg/m^3) and A is wind turbine blade sweep area (m^2), $f(v)$ is pdf of wind speed.

The mean power density of time series wind data, denoted as(P_{REF}) is estimated as follows:

$$P_{REF} = \frac{1}{2}\rho A \frac{1}{n}\sum_{i=1}^{n} v_i^3 \tag{8}$$

By using Equations (7) and (8), P_D values, corresponding to the WD (P_{WD}) and the MWD (P_{MWD}), P_{REF} values are calculated for Bandırma and Kirecburnu. Furthermore, the obtained values of P_{WD} , P_{MWD} , P_{REF} andPDE, corresponding to the WD (PDE_{WD}) and the MWD(PDE_{MWD}), are given for Bandırma and Kirecburnu in Table 6 and Table 7, respectively

Table 6. Wind power density error (%) for the WD and the MWD for Bandırma.

	PREF	PWD	PMWD	PDEWD	PDEMWD
Jan.	136.117	164.690	111.075	20.991	18.397
Feb.	132.625	144.659	106.094	9.073	20.004
Mar.	163.011	323.081	150.484	98.196	7.684
Apr.	67.437	76.863	62.992	13.978	6.592
May	75.909	115.242	69.495	51.815	8.449
Jun.	50.576	82.127	48.846	62.382	3.420
Jul.	54.310	116.539	50.868	114.581	6.337
Aug.	100.924	170.024	96.003	68.467	4.876
Sept.	154.590	387.603	148.157	150.729	4.161
Oct.	198.737	301.380	146.453	51.647	26.308
Nov.	54.272	65.338	47.479	20.390	12.514
Dec.	127.392	103.350	87.088	18.872	31.637
Year	110,415	144,827	105,965	31,166	4.030

Table 7. Wind power density error (%) for the WD and the MWD for Kireçburnu.

	PREF	PWD	PMWD	PDEWD	PDEMWD
Jan.	50.299	102.035	45.808	102.856	8.928
Feb.	25.738	29.765	18.820	15.645	26.878
Mar.	10.243	15.840	10.199	54.644	0.426
Apr.	8.348	16.306	8.276	95.331	0.862
May	7.363	13.613	7.322	84.871	0.565
Jun.	6.058	13.040	6.042	115.241	0.257
Jul.	18.202	41.518	17.579	128.094	3.424
Aug.	8.886	17.712	9.214	99.324	3.700
Sept.	12.685	28.098	12.754	121.507	0.544
Oct.	27.903	66.658	28.555	138.889	2.335
Nov.	7.596	15.992	7.486	110.532	1.448
Dec.	10.657	14.837	10.128	39.220	4.960
Year	16.320	24.932	15.788	0.527	0.033

It can be observed from the Table 6 that the MWD can yield less error values than the WD for the most of the considered data. Furthermore, it is deduced from the Table 7 that the MWD provides a substantial improvement over the WDin estimating wind power for all months and whole year. Thus, the results of analysis show that the MWD is superior to the WD for estimating wind power.

Also, Figure 9-11 demonstrate the fitting ability of the MWD versus the WD for wind speed data observed from Bandırma. Similarly, Figure 12-14 illustrate the fitting ability of the considered distributions for wind speed data of Kireçburnu. These figures provide visual information that the MWD exhibits a better fit than the WD.

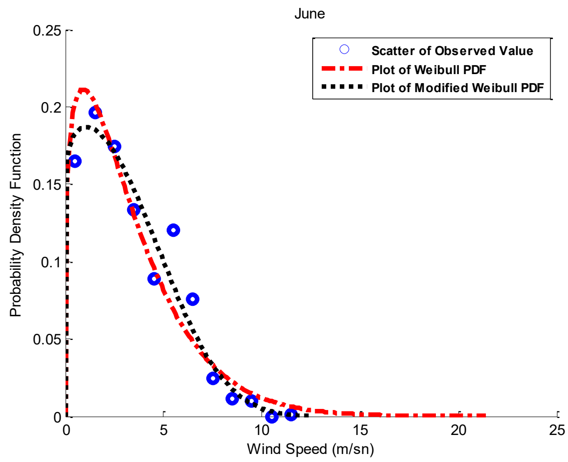

Figure 9. Scatter and pdf graphs of the WD and the MWD for June data observed from Bandırma.

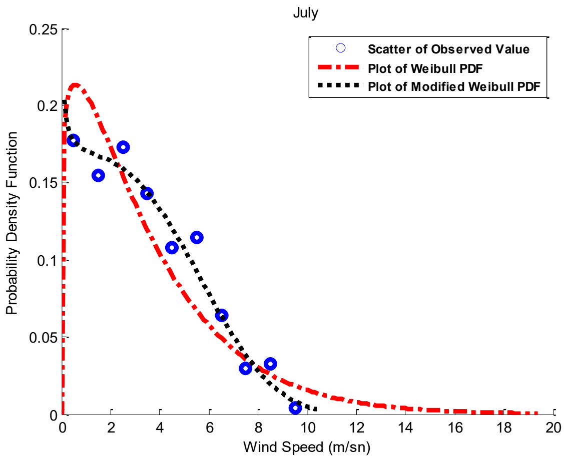

Figure 10. Scatter and pdf graphs of the WD and the MWD for July data observed from Bandırma.

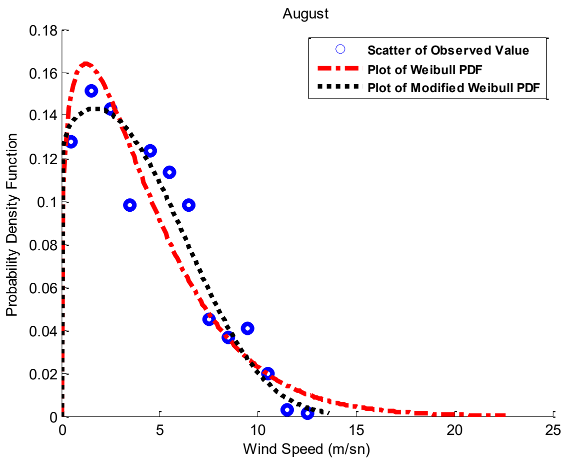

Figure 11. Scatter and pdf graphs of the WD and the MWD for August data observed from Bandırma.

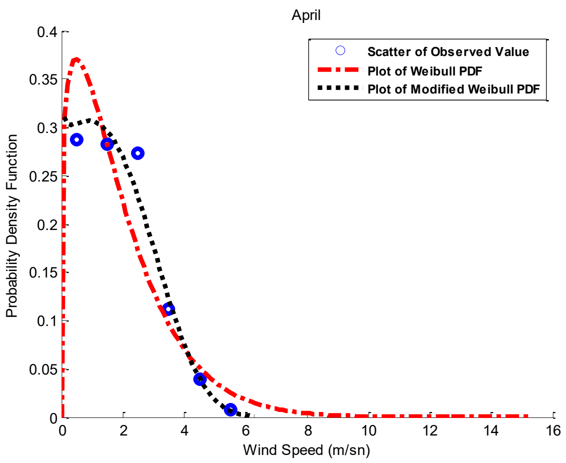

Figure 12. Scatter and pdf graphs of the WD and the MWDfor April data observed from Kireçburnu.

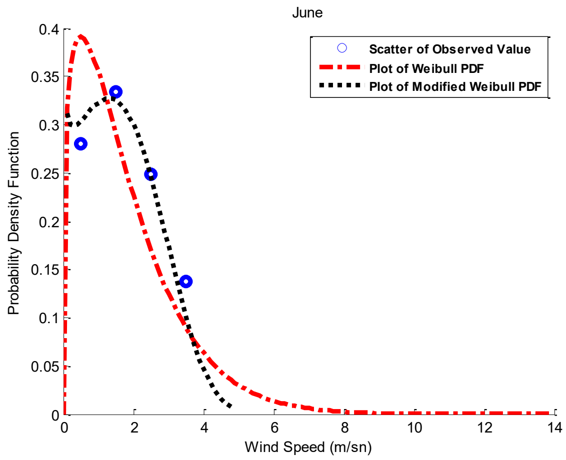

Figure 13. Scatter and pdf graphs of the WD and the MWD for June data observed from Kireçburnu.

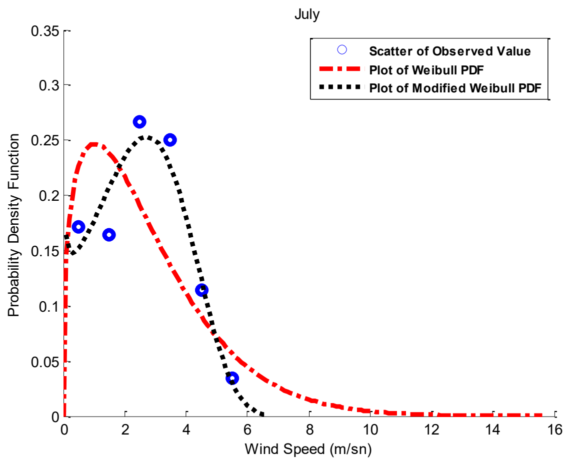

Figure 14. Scatter and pdf graphs of the WD and the MWD for July data observed from Kireçburnu.

4. CONCLUSIONS

It is well-known that wind energy potential can be estimated by using wind speed distribution. For this reason, finding a suitable and accurate wind speed distribution is crucial in wind energy applications. In this context, two parameters Weibull distribution (WD) is the well-accepted distribution in wind energy studies. However, the WD does not exhibit a good fitting for all wind speed cases measured at different geographical locations throughout the world.

Thus, the main objective of this paper is to proposethe modified Weibull distribution (MWD) to describe wind speed distribution and also to evaluate its capability in modeling wind speed data. Because it is known that the MWDismore flexible than the WDand nests the WDas a special case.

For this purpose, the MWD and the WD are compared on real wind speed data measured in two different regions of Turkey. The fitting accuracy of the proposed the MWD is judged from different model selection criteria commonly used in wind energy literature. Also, power density error (PDE) is used to evaluate the capability of distribution in estimating wind power.

The results obtained in this study can be summarized as follows.

- It is found that the MWD outperforms the WD for the most of the considered wind speed cases in terms of criteria.
- It is observed that the MWD has smaller PDE values than the WD for the most of the considered wind speed data.
- It is deduced from scatter and pdf graphs that the MWD provides a more fitting ability than the WD for the most of considered data.
- Consequently, the MWDcan be an alternative to use for assessment of wind energy potential.
- As a further study, we aim to compare the performance of the MWD with the different flexible distribution for modeling wind speed data.

5. ACKNOWLEDGMENT

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6. REFERENCES

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