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A Hierarchical Linear Models Approach to Explaining Reflective Thinking Skill Through Problem Solving[1]

Problem Çözmeye Yönelik Yansıtıcı Düşünme Becerisinin Açıklanmasında Hiyerarşik Lineer Modeller Yaklaşımı[1]

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ABSTRACT: The purpose of this study is to identify the student and teacher level factors that explain the reflective thinking skills through problem solving of elementary school pupils. The research was conducted in public elementary schools in three Van districts: Pekyolu, Tuşba, and Edremit. The study's sample consists of 52 mathematics teachers and 1126 elementary school pupils. "Participant Information Form," "Reflective Thinking Ability Through Problem Solving Scale," and "Mathematics Problem Solving Attitude Scale" are data gathering tools for pupils. Teachers' data was gathered using the "Participant Information Form," the "Reflective Thinking Tendency Scale for Teachers and Preservice Teachers," and the "Scale for Learner Autonomy Support." According to the results, it is found that the 90 % of variance in dependent variable can be explained by student level variables. Teacher level variables could explain 10% of outcome variance. The gender of students, receiving private tuition, book reading frequency, time devoted to studying mathematics, and attitude through problem solving are significant factors that can predict the dependent variable. Moreover, students' reflective thinking ability can be significantly influenced by teacher-level factors, such as teachers' knowledge of problem-solving steps and their tendency for reflective thinking. The authors recommend education or seminars that develop teachers' reflective thinking tendency and problem-solving courses be provided in mathematics teaching undergraduate programs. Students should be supported in gaining reading habit and activities that have positive impact on the students' problem-solving attitude.

Keywords: Reflective thinking, hierarchical linear models, problem solving

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ÖZ: Bu çalışmanın amacı, ilkokul öğrencilerinin problem çözme yönelik yansıtıcı düşünme becerilerini açıklayan öğrenci ve öğretmen düzeyindeki faktörleri belirlemektir. Araştırma, Van'ın üç ilçesindeki (İpekyolu, Tuşba ve Edremit) devlet ilköğretim okullarında yürütülmüştür: Araştırmanın örneklemini 52 matematik öğretmeni ve 1126 ilkokul öğrencisi oluşturmaktadır. "Katılımcı Bilgi Formu", "Problem Çözmeye Yönelik Yansıtıcı Düşünme Becerisi Ölçeği" ve "Matematik Problemi Çözme Tutum Ölçeği" öğrenciler için veri toplama araçlarıdır. Öğretmenlerin verileri “Katılımcı Bilgi Formu”, “Öğretmen ve Öğretmen Adayları için Yansıtıcı Düşünme Eğilimi Ölçeği” ve “Öğrenen Özerkliğini Destekleme Ölçeği” kullanılarak toplanmıştır. Sonuçlara göre bağımlı değişkendeki varyansın %90'ının öğrenci düzeyi değişkenler tarafından açıklanabildiği belirlenmiştir. Öğretmen düzeyindeki değişkenler, bağımlı değişkendeki varyansının %10'unu açıklayabilmektedir. Öğrencilerin cinsiyeti, özel ders alma durumu, kitap okuma sıklığı, matematiğe ayrılan zaman ve problem çözmeye yönelik tutum bağımlı değişkeni yordayan önemli faktörlerdir. Ayrıca, öğretmenlerin problem çözme adımlarına ilişkin bilgi düzeyi ve yansıtıcı düşünme eğilimi gibi öğretmen düzeyindeki faktörler öğrencinin yansıtıcı düşünme yeteneğini manidar bir biçimde yordamaktadır. Yazarlar, matematik öğretmenliği lisans programlarında öğretmenlerin yansıtıcı düşünme eğilimlerini ve problem çözme derslerini geliştirecek eğitim veya seminerlerin verilmesini önermektedir. Öğrencilere okuma alışkanlığı kazandırılması ve öğrencilerin problem çözme tutumlarına olumlu etkisi olan etkinlikler desteklenmelidir.

Anahtar sözcükler: Yansıtıcı düşünme, hiyerarşik lineer modeller, problem çözme

1. INTRODUCTION

Reflective thinking, which is attributed importance in the pragmatism approach, is a thinking style that is defined by prominent American philosopher John Dewey (1933). Reflecting thinking is a concept open to improvement and used frequently in educational sciences in recent years (Ünver, 2003). Dewey (1933) defines reflective thinking as to comprehend knowledge, and inferences deduced from the knowledge actively, continuously and carefully on a basis supporting the knowledge. It can be said that the definition points to two aspects of reflective thinking. Moreover, the definition examines the current conditions that provide support and identifies the sources of this support. It then outlines the results and inferences that can be drawn from these conditions and sources. He acknowledges that reflection differs from other modes of thinking in two ways. First, reflection is characterized by mistrust, hesitancy and mental enforcement. Second, reflection prompts people to do convenient searches, queries that may alleviate distrust and hesitancy and to search for resources that can aid in problem solving (Kember, 2008). Thus it can be said that the definition of reflective thinking emphasizes the need of active, continuous, and careful thinking in all stage of problem solving (Rodgers, 2002). According to Boyd and Fales (1983), reflection is an introspective process initiated upon encountering a problem. This process aids individuals in generating or elucidating meaning and expanding their conceptual framework. Epstein (2003) asserts that most educators concentrate on the memorization capability of young students. Although, those educators expect students to remember old learnings, reflective thinking is not recitative repetition or recall of knowledge. The author thinks reflective thinking is a recall process with analysis. When students are exposed to activities that they could employ reflection, they can go beyond reporting activities they participated in. Through reflection, they can recognize what they learnt during the activities, what they are interested in, and determine how they can expand their experiences. Reflective thinking is a sort of thinking which is activated when faced with a problem. Thus, reflective thinking can be observed better during a problem-solving stage (Kızılkaya & Aşkar, 2009). Hegedus (2002) describes reflection types that emerged during problem solving in three categories. Those are:

I. Forward reflection

II. Backward reflection

III. A-temporal reflection

Forward reflection is looking forward into a solution of a problem and how the solution can be developed by predicting possible outcomes based on solution so far or through self-experiences. Backward reflection, on the other hand, is to control and confirm chosen algebraic and geometric structures in a solution at any stage of the process. In other words, forward reflection refers to thinking on why these structures were chosen and how they contribute to the effort of development of the solution. A-temporal reflection is the process of simultaneous or undegrading checking which contributes to the coherence or development of the solution. It can be deduced that reflective thinking is closely related to problem solving stages and one of the objectives that formal education aims to instill in students (Hegedus, 2002).

The characteristics that determine reflective thinking capacity may be investigated on two levels. The first level is the student level, which comprises variables that are directly tied to students. The latter is at the teacher level and includes indirect elements that explain pupils' reflective thinking capacity. Reflective thinking is crucial for both students and teachers. For teachers, it's vital for their professional development because it helps them regularly assess themselves and recognize their strengths and weaknesses. By reflecting on their teaching practices, teachers can continuously monitor, evaluate, and improve their methods. This means teachers who engage in reflective thinking are expected to creatively plan their lessons and activities, ultimately leading to better education quality (Orakcı et al., 2020). At the same time, teachers who use reflective thinking to understand the details of their students' activities are more effective in meeting their students' needs. Moreover, they can use teaching itself as a way to learn, allowing them to improve over time by learning from their mistakes (Barnhart & van Es, 2015). However, when examining the existing literature, it becomes apparent that studies focusing on reflective thinking skills often either involve only students or exclusively target teachers. As a result, there is a lack of research exploring how teachers' tendencies towards reflective thinking and other demographic and affective variables affect students' reflective thinking abilities.

The purpose of this study is to identify student-level and teacher-level variables that can predict reflective thinking through problem-solving skills in elementary school students. The main goal of this study is to answer the following question: "Which student level (Level-I) and teacher level (Level-II) factors may describe reflective thinking through problem solving capacity of elementary school students significantly?" To address this overarching goal, the study examines three specific sub-questions. Those are as follows:

I. Does reflective thinking through problem solving ability differentiate between 2nd level unites (teachers)?

II. Do 1th level (student) variables (gender, age, taking an elective math course, receiving private tuition, education level of father and mother, the average income of family, reading frequency per week, time allocated to study mathematics, and attitude on problem solving) predict dependent variable significantly.?

III. Do 2nd level (teacher) variables (gender, age, length of service, support level of learner autonomy, instruction of elective mathematics course, taking graduate education, taking a problem solving course, knowledge in problem solving steps and reflective thinking tendency level) predict reflective thinking ability through problem solving of students?

2. METHOD

2.1. Research Design

In this research we looked for the independent variables at the student and teacher levels that significantly predict reflective thinking through problem-solving skill. For this study, it can be said that the study has a descriptive-correlational design since the independent variables at the student and teacher level predict students' reflective thinking skills without manipulating any variable (Fraenkel et al., 2012).

2.2. Population and Sample

The study's target population consists of students in the sixth, seventh, and eighth grades from three districts in Van (İpekyolu, Tuşba, and Edremit). Researchers sampled from both student and teacher populations because hierarchical linear models require hierarchical data. According to Maas and Hox (2005), hierarchical linear models can yield reliable parameter estimates with at least 50 second-level units. Therefore, the researchers sampled 1,126 students nested within 52 teachers from the defined districts. For this, we employed stratified sampling, a random sampling approach.

2.3. Data Collection Tools and Analysis

Students who share the same classrooms in the same schools tend to be similar in terms of factors influencing accomplishment, such as teacher qualities, socioeconomic level, and age (Yıldırım, 2012). Many studies in the literature advocate investigating such multilevel data structures with hierarchical linear models (HLM). Hierarchical linear models, when dealing with hierarchical data, are a collection of regression analysis approaches that can estimate group-based intercept and slope parameters (Gelman & Hill, 2006). In most educational or social science research, nested or hierarchical data samples are used. Samples collected using stratified or cluster sampling procedures may violate the independence of observations. Consequently, analyzing such data with HLM is more convenient as it addresses the problem of dependent observations (Arnold, 1992; Yıldırım, 2012), provides a more accurate estimate of standard error (Gelman & Hill, 2006; Raudenbush & Willms, 1991) and tackles challenges associated with aggregation and disaggregation (Hox, 2010).

The data collection tools for students are demographic variable forms for participants, the Reflective Thinking Ability Through Problem Solving Scale (RTPSS) developed by Kızılkaya and Aşkar (2009) and the Mathematics Problem Solving Attitude Scale (MPSAS) developed by Çanakçı and Özdemir (2011). The Reflective Thinking Ability Through Problem Solving scale comprises 14 Likert-type items factorized under three factors: questioning, reasoning, and evaluation. However, the subscale scores have not been calculated, and only the total score has been used as the dependent variable for the students. The other tool, the Mathematics Problem Solving Attitude Scale (MPSAS), consists of 19 Likert-type items clustered into two sub-factors: "enjoyment" and "teaching." Similar to the previous scale, the subscale scores have not been calculated; only the total score has been used as the independent variable representing students' attitudes towards solving mathematical problems. The internal reliability of the RTPSS is 0.83, while the reliability coefficient of the MPSAS is 0.77 for the current study.

The data for teachers were collected using the Reflective Thinking Tendency Scale for Teachers and Teacher Candidates (RTTS) developed by Semerci (2007), the Scale for Learner Autonomy Support (SLAS) developed by Oğuz (2013), and a demographic variables form. The RTTS comprises 35 Likert-type items and has an internal consistency coefficient of 0.92. For the current study, the SLAS exhibits an internal reliability coefficient of 0.87. None of the sub-factors of either scale were utilized in this study. Instead, total scores obtained from both scales were employed as independent variables attributed to teachers. The data was analyzed with SPSS and HLM 7 (Bryk et al., 2010) softwares.

Researchers employed lowercase letters for Level-I variables and uppercase letters for Level-II variables to prevent confusion. Level-I variables were coded as follows: gen = gender, age = age, emc = taking elective mathematics course, rpt = receiving private tuition, fel = father's education level, mel = mother's education level, afi = average family income, brf = book reading frequency, tsm = time devoted to studying mathematics, aps = attitude towards problem solving. The abbreviations for Level-II variables are as follows: GEN = gender, AGE = age, LES = length of service, IMC = instruction of elective mathematics course, SOL = taking problem solving course, PSS = knowledge in problem solving steps, TGE = taking graduate education, SLA = support level of learner autonomy, RTT = reflective thinking tendency level.

A number of scholars have proposed various strategies to rescale the level-1 predictors in order to make intercepts more interpretable. "Centering" refers to the rescaling of the level-l predictors, which has three basic options: (1) raw metric approaches, in which no centering occurs and level-1 predictors retain their original metric, (2) grand mean centering, in which the grand mean is subtracted from each individual's predictor score, and (3) group mean centering, in which the group mean is subtracted from each individual's predictor score. The intercept in grand mean centering provides the predicted level of result for a person with an "average" level on the predictor (Hofmann, 1997; Hox and Roberts 2011; Leeuw & Meijer, 2008). For this study grand mean centering was used to rescale Level-I variables.

According to the research problems, three hierarchical linear models were fitted. The first model utilized a one-way ANOVA with a random effect. This model did not include any explanatory variables. Equation 1 represents the mathematical expression of this model. It helps determine the proportion of variance explained by both levels, even though there are no independent variables in the model.

$$Level\ 1(Student)\ Model: \quad Y_{ij} = \beta_{0j} + r_{ij}$$
$$Level\ 2\ (Teacher)\ Model \quad \beta_{0j} = \gamma_{00} + u_{0j} \qquad 1$$

In Equation 1, Y_{ij} is the outcome variable for student *i* nested in teacher *j*. β_{0j} is the mean of students in teacher *j*. The Level-I error term r_{ij} is the difference in outcome of student *i* from the mean of students of teacher *j*. The grand mean of all students nested in all teachers is shown by γ_{00}. And finally, Level-II error term u_{0j} is the difference between the grand mean and mean of students of teacher *j*.

To investigate the effects of Level-I explanatory variables we fitted random coefficient regression model (RCRM). The mathematical equation regarding this study can be reached in Equation 2. At the equation β_{0j} is intercept for students nested in teacher *j*. The symbols β_{1j} to β_{10j} are coefficients for each explanatory Level-I variable.

Level-I(Student) Model:

$$Y_{ij} = \beta_{0j} + \beta_{1j} * gen + \beta_{2j} * age + \beta_{3j} * emc + \beta_{4j} * rpt + \beta_{5j} * fel + \beta_{6j} * mel + \beta_{7j} * afi + \beta_{8j} * brf + \beta_{9j} * tsm + \beta_{10j} * aps + r_{ij}$$

Level-II (Teacher) Model:

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + u_{0j} \\ \beta_{1j} &= \gamma_{10} + u_{1j} \\ \beta_{2j} &= \gamma_{20} + u_{2j} \\ \beta_{3j} &= \gamma_{30} + u_{3j} \\ \beta_{4j} &= \gamma_{40} + u_{4j} \\ \beta_{5j} &= \gamma_{50} + u_{5j} \\ \beta_{6j} &= \gamma_{60} + u_{6j} \\ \beta_{7j} &= \gamma_{70} + u_{7j} \\ \beta_{8j} &= \gamma_{80} + u_{8j} \\ \beta_{9j} &= \gamma_{90} + u_{9j} \\ \beta_{10j} &= \gamma_{100} + u_{100j} \qquad 2 \end{aligned}$$

Finally, we fitted means as outcome model (MOAM) to determine the effects of Level-II variables. The equation of the model can be seen in Equation 3.

Level-I(Student) Model:

$$Y_{ij} = \beta_{0j} + r_{ij}$$

Level-II (Teacher) Model:

$$\boldsymbol{\beta_{0j} = \gamma_{00} + \gamma_{01} * (GEN_j) + \gamma_{02} * (AGE_j) + \gamma_{03} * (LES_j) + \gamma_{04} * (IMC_j) + \gamma_{05} * (SOL_j) + \gamma_{06} * (PSS_j) + \gamma_{07} * (TGE_j) + \gamma_{08} * (SLA_j) + \gamma_{09} * (RTT_j) + u_{0j}} \quad \textbf{3}$$

3. FINDINGS

3.1. Findings Regarding the First Problem

The first research problem was analyzed with one-way ANOVA with random effect and the findings are provided in Table 1. It can be seen that the fixed effects of the one-way ANOVA with random effect analysis significantly predict (t=49.89, p<.05) the dependent variable. This means that without any explanatory variable reflective thinking tendency of a typical student is 49.89. After determining the significance of the fixed effect, we get the random effects of the model and summarized the findings in Table 2.

Table 1: *Fixed Effects of One-Way ANOVA With Random Effects Model*

Fixed Effects	Coefficients	Se	t	df	p
Mean, γ_{00}	49.89	0.53	94.99	51	0. 00*
*p<.05					

Table 2: *Random Effects of One-Way ANOVA with Random Effects Model*

Random Effects	Sd	σ^2	df	χ^2	p
Level 2 Error (u_{oj})	3.26	10.61	51	189.76	0.00*
Level 1 Error (r_{oj})	9.27	85.94			
*p<.05					

With hierarchical linear models, a researcher can handle variation in dependent variables at two levels. According to the results, the variation can be explained by the second level units significantly (

χ^2=189.76, p<.05). We used Equation 1 to compute the intra-class correlation coefficient to determine the percentage of variation accounted for by the second level components. A 10% fraction of the difference in reflective thinking via problem solving competence may be explained by teacher-related factors. While student-level independent factors can explain the remaining 90% of the variance. This finding shows that, because the amount of variance explained by second level variables is 10%, it is more convenient to examine reflective thinking skills at two levels.

$$Between - group\ variation\ ratio =* \rho = \frac{\tau_{00}}{\tau_{00} + \sigma^2} = \frac{10.61}{10.61 + 85.94} = 0.10 \quad (1)$$

$$Within - group\ variation\ ratio = 1.00 - 0.10 = 0.90$$

*ρ= intra-class correlation coefficient

3.2. Findings Regarding the Second Problem

We conducted random coefficient regression model (RCRM) analysis to detect which Level-I variables explain student level variability significantly. The details of fixed effects derived from the fitted model result can be seen in Table 3.

Table 3: *Fixed Effects of Random Coefficient Regression Model*

Fixed Effects	Coefficients	Se	t	p
Mean, γ_{00}	58.64	1.62	36.13	0.00*
gen, γ_{10}	-1.23	0.50	-2.46	0.02*
age, γ_{20}	0.11	0.30	0.38	0.70
emc, γ_{30}	-0.97	0.53	-1.84	0.07
rpt, γ_{40}	-2.03	0.60	-3.39	0.00*
fel, γ_{50}	0.24	0.28	0.84	0.41
mel, γ_{60}	0.09	0.27	0.33	0.75
afi, γ_{70}	-0.34	0.41	-0.83	0.41
brf, γ_{80}	-1.52	0.36	-4.25	0.00*
tsm, γ_{90}	0.42	0.07	5.98	0.00*
aps, γ_{100}	0.37	0.03	13.27	0.00*

*p<.05, gen=gender, age=age, emc= taking elective mathematics course, rpt= receiving private tuition, fel=father education level, mel=mother education level, afi=average family income, brf= book reading frequency, tsm=time devoted to studying mathematics, aps=attitude towards problem solving

Table 3 shows that the age of the students (t=0.38, p>.05), the father's educational level (t=0.84, p>.05), the mother's educational level (t=0.33, p>.05), and the family's average income (t=-0.83, p>.05) do not predict reflective thinking skills through problem-solving. However, students' gender (t=-2.46, p.05), whether they receive private instruction (t=-1.84, p>.05), how often they read (t=-4.25, p.05), how much time they spend studying math after school (t=5.98, p.05), and their attitude toward solving math problems (t=13.27, p.05) significantly predict the dependent variable. Female students have 1.23 points higher score than male students. In other words, female students are more reflective thinkers than male pupils. Private instruction has a beneficial impact on reflective thought and may raise reflective thinking by 2.03 points. The table demonstrates that frequent book reading adversely predicts reflective thinking. Reflective thinking decreases by 1.52 units for every unit increase in book reading frequency. This is due to how the categories of book reading frequency were coded. As a result of how we classified the categories: One book per week (1), one book per month (2), one book per semester (3), and one book or fewer per year (4). Even though the variable's estimate is negative, we can still conclude that as students read more books, their ability to reflect improves. The ability of students to think critically and reflectively has also benefited from their extracurricular math study time. An increase in mathematical study time may result in a 0.42-unit increase in reflective thinking abilities. Last but not least, students' perspectives on problem solving positively influence reflective thinking abilities. Students who score 0.37 points higher on the reflective thinking ability scale have a one-unit more positive attitude toward problem solving. Table 4 displays the model's random effects.

Table 4: *Random Effects of Random Coefficients Model*

Random Effects	Sd	σ^2	χ^2	p
Level 2 Error (u_{oj})	4.31	18.55	20.60	>.50
gen, u_{1j}	1.63	2.65	38.20	0.08
age, u_{2j}	1.09	1.18	41.45	0.04*
emc, u_{3j}	1.94	3.77	20.75	>.50
rpt, u_{4j}	2.26	5.12	18.77	>.50
fel, u_{5j}	1.20	1.43	38.01	0.08
mel, u_{6j}	0.75	0.56	18.17	>.50
afi, u_{7j}	1.61	2.59	51.82	0.00*
brf, u_{8j}	0.94	0.88	44.05	0.02*
tsm, u_{9j}	0.20	0.04	27.44	0.44
aps, u_{10j}	0.11	0.01	41.65	0.05*
Level 1 Error (r_{oj})	7.56	57.09		

In Table 4. it can be seen that the variance of age of students (χ^2=41.45, p<.05), average income of family (χ^2=51.82, p<.05), book reading frequency (χ^2=44.05, p<.05) and attitude towards solving mathematics problems (χ^2=41.65, p<.05) differentiate across second level units. The variances of the other factors are assumed to be equal between groups. To calculate the explanatory power of student-level variables in the current model, Equation 2 was employed. This equation calculates the percentage reduction in variance at Level-I (R_1^2). To get R_1^2, Level-I variance ($r_{oj} = 57.09$) of the recent model subtracted from null model Level-I variance ($r_{oj} = 85.94$). Then the result is divided by the null model Level-I variance ($r_{oj} = 85.94$).

$$R_1^2 = \frac{\sigma_{(ANOVA)} - \sigma_{(RERM)}}{\sigma_{(ANOVA)}} \qquad R_1^2 = \frac{85{,}94 - 57{,}09}{85{,}94} = 0{,}34 \qquad (2)$$

According to calculations in Equation 2, it can be concluded that significant variables which are students' gender (t=-2.46, p<.05), whether they receive private instruction (t=-1.84, p>.05), how often they read (t=-4.25, p.05), how much time they spend studying math after school (t=5.98, p.05), and their attitude toward solving math problems (t=13.27, p.05) can explain 34% of variance at Level-I. It may be deduced from the result that more Level-I variables that are related to reflective thinking can increase the explanation rate.

3.2. Findings Regarding the Third Problem

Researchers conducted a means-as-outcome model to identify teacher-level factors that significantly predict reflective thinking (MAOM). For this model, researchers focused exclusively on Level-II elements, including nine variables in the study and excluding Level-I variables. These variables are: gender (GEN), age (AGE), length of service (LES), instructing of an elective mathematics course (IMC), taking a problem-solving course (SOL), knowledge in problem solving steps (PSS), enrolling in graduate education (TGE), support level of learner autonomy (SLA), and reflective thinking tendency level (RTT). Table 5 presents the model's fixed effects.

Table 5: *Fixed Effects of Means as Outcome Model*

Fixed Effects	Coefficients	Se	t	p
Mean, γ_{00}	54.05	9.32	5.80	0.00
GEN, γ_{10}	-0.72	0.97	-0.74	0.46
AGE, γ_{20}	0.07	0.33	0.22	0.83
LES, γ_{03}	0.15	0.36	0.42	0.68
IMC, γ_{04}	-0.64	1.12	-0.57	0.57
SOL, γ_{05}	-0.27	0.97	-0.27	0.79
PSS, γ_{06}	-2.53	1.00	-2.52	0.02*
TGE, γ_{07}	1.32	1.22	1.08	0.29
SLA, γ_{08}	0.00	0.07	0.04	0.97
RTT, γ_{09}	0.19	0.04	4.81	0.00*

*p<.05, GEN=gender, AGE=age, LES= length of service, IMC= instruction of elective mathematics course, SOL= taking problem solving course, PSS= knowledge in problem solving steps, TGE= taking graduate education, SLA=support level of learner autonomy, RTT= reflective thinking tendency level

In Table 5. it is observed that, gender of teachers (t=-0.74, p>.05), age ((t=-0.22, p>.05),), length of service (t=0.42, p>.05), instruction of elective mathematics course (t=-057, p>.05), taking problem solving course (t=-027, p>.05), taking graduate education (t=1.08, p>.05), and support level of learner autonomy (t=0.04, p>.05) do not predict dependent variables significantly. However, teachers' knowledge of problem solving steps (t=-2.52, p<.05) and reflective thinking tendency level (t=4.81, p<.05) are significant in the prediction of reflective thinking skills. According to the results, students of teachers who report having a high level of knowledge in problem-solving steps perceive themselves as having lower reflective thinking skills through problem solving. The difference between these students and students of teachers who report having a medium level of problem-solving step knowledge is 2.53 points. It can be seen that teachers' reflective thinking tendency level can play a positive role in students reflective thinking skills. One unit change in this variable can leads to 0.19 points change in students reflective thinking skills. Random effects of the model are presented in Table 6.

Table 6: *Random Effects of Means as Outcome Model*

Random Effects	sd	σ^2	χ^2	p
Level 2 Error (u_{oj})	2.35	5.51	99.96	0.00*
Level 1 Error (r_{oj})	9.27	85.99		

*p<.05

$$R_1^2 = \frac{\sigma_{(ANOVA)} - \sigma_{(MAOM)}}{\sigma_{(ANOVA)}} \qquad R_1^2 = \frac{10.61 - 5.51}{10.61} = 0.48 \qquad (2)$$

It can be seen that Level-I variance is 85.99 in Table 6. The Level-I variance of the null model and this estimate are quite similar. However, the variance of the random coefficient regression model is significantly lower because the MAOM model does not include a Level-I explanatory variable. As a result, the Level-I variance results are roughly in line with researchers' expectations, while a significant reduction in Level-II variance was anticipated. The level II variance is predicted to be 5.51, which is less than the variance in base and RCRM, as expected. Equation 2 was applied by the authors to determine this reduction percentage. The random coefficient of the MAOM model differs from that of the ANOVA model by 0.48, leading to a 48% reduction in unexplained Level-II related variance when the Level-II coefficient was included.

4. DISCUSSION and RESULTS

The main goal of this study is to determine the prediction status of student (Level-I) and teacher (Level-II) level factors of reflective thinking skills towards problem solving. We ran three hierarchal linear models to identify the effect of the independent variables separately. According to the null model (random coefficient ANOVA) result we could observe that reflective thinking skill of elementary students differs in terms of the mathematics teachers who teach them. We reached an unignorably amount of variance that can be explained by teacher related factor.

The variance in reflective thinking skills among students may indeed be influenced by the discussion and reflection settings facilitated by teachers. This result aligns with findings from many studies in the literature, indicating that teachers who promote student participation in discussion and reflection activities tend to have students with higher levels of reflective thinking skills (Hassan et al., 2016; Töman, 2017; Yeşilbursa, 2011). This finding highlights the critical role of interactive teaching methods in developing students' reflective thinking skills. By promoting an environment where students are encouraged to participate in meaningful discussions and reflect on their learning experiences, teachers can significantly enhance their students' reflective thinking capabilities. This approach not only improves students' problem-solving skills but also prepares them for lifelong learning and adaptation in various contexts. Studies show that reflection skills and evaluation tasks are linked. A systematic review of reflective writing in education found positive effects on student growth in reflection, learning, reflection skills, self-assessment, problem solving (Woldt & Nenad; 2021). A study on the need for a reflective module for elementary school students' literacy and numeracy skills found a correlation between reflective thinking and assessment activities in students (Rakhmawati & Mustadi, 2021). Lipton and Hubble (1998) asserts that there is a close relationship between reflection skills and assessment activities. According to them, the frequency of self and peer assessment may play a role in awakening reflective thinking. Therefore, it is reasonable to expect that teachers who incorporate more peer and self-assessment practices in their measurement and assessment approaches can observe greater reflection skills in their students (Groom & Maunonen-Eskelinen, 2006). Peer and self-assessment allow students to critically evaluate their own and others' work, fostering a deeper understanding of the subject matter and enhancing their ability to reflect on their learning processes. Consequently, these practices can lead to improved reflective thinking skills, better problem-solving abilities, and a more comprehensive grasp of the content being taught. Thus, it is appropriate for teachers to place greater emphasis on measurement and evaluation practices that support reflective skills.

We conducted random coefficient regression model to analyze student level independent variables' effects on reflective thinking skills. The model indicated that the age of students, the father's educational level, the mother's educational level, and the family's average income do not predict significantly reflective thinking skills towards problem-solving. The results about age and parental education level of students are similar to the findings of the study conducted by Saygılı and Atahan (2014). They found that age and parental education level of talented children do not affect reflective thinking. Aydın and Çelik (2013) declare in their study, which explores the effects of factors that explain social science teachers' reflective thinking, that grade level and parental education do not predict reflective thinking significantly. However, there are also studies indicating that the educational level of the parents has a significant impact on students' levels of reflective thinking skills (Can, 2015). There are some studies which reached parallel findings about average income with the recent study. For instance, Ceylan (2014), Kırnık (2010), and Gedik et al. (2014) found no effect of average income on reflective thinking. Additionally, students' propensity to read books positively predicts the dependent variable. Reflective thinking skills improve as reading frequency rises. Similar findings about book reading and reflective thinking were provided by Ceylan (2014), who examined the levels of reflective thinking in pupils. We can infer that reading books does not only improve one's ability to communicate or comprehend, but it also immediately enhances mathematical abilities and reflective thinking. This finding suggests that reading promotes cognitive skills beyond language proficiency, influencing how individuals approach problem-solving and self-reflection. Thus, engaging in reading activities can contribute significantly to overall cognitive development, enhancing both academic and analytical capabilities across various domains. Once more, we discovered in a recent study that attitude toward solving mathematics problems significantly predicts reflective thinking abilities. This conclusion is parallel with the finding of Demirel et al. (2015). This finding is not surprising because reflective thinking prompts individuals to consider a strategy and assess it to make appropriate decisions in problem-solving (Gencel & Saracaloğlu, 2018; Hatton & Smith, 1995; Hayden & Chiu, 2015). Another assertion made by Walle and John (1998) can be used to explain the association between reflective thinking abilities and attitude toward approaching mathematics problems. They contend that the relationship between time spent on problem-solving and satisfaction from it is unbreakable. People who enjoy solving problems tend to work harder to find a solution. However, those with a negative attitude often give up after their first or second setback.

We ran means as outcome model to get effects of Level-II factors that can affect reflective thinking. According to the result we conclude that gender and age of teachers, length of service, instruction of elective mathematics course, taking problem solving course, taking graduate education, and support level of learner autonomy do not predict dependent variables significantly. On the other hand, teachers' knowledge of problem solving steps and reflective thinking tendency levels have a significant effect in explaining of variation of reflective thinking skills. An important finding of this study is that the reflective thinking tendency of teachers predicts students' reflective thinking skills positively. That is to support the development of reflective skills in students, it is essential for teachers to engage in reflective practices themselves. Several studies have highlighted the significance of teacher reflection in enhancing students' ability to reflect (Kheirzadeh & Sistani, 2018; Wopereis et al., 2010). Duban and Yanpar-Yelken (2010) confirm this result and assert that teachers who do more reflection, enforce students to present more reflective skills. Additionally, they claim that encouraging students' reflective thinking might help them develop high-level cognitive abilities like analysis, synthesis, and assessment and lead to a more effective community. Additionally, they claim that encouraging students' reflective thinking might help them develop high-level cognitive abilities like analysis, synthesis, and assessment and lead to a more effective community.

3.1. Recommendation

The findings allow for the following recommendations:

1. It is suggested that courses or seminars addressing these matters should be included in undergraduate teachers training programs, as it has been observed that variables associated with teachers influence students' reflective thinking skills.

2. From the beginning of their schooling, teachers and families should encourage pupils to make reading a habit. Although some teachers make reading required, the goal should be to instill a love of reading in kids.

3. Students' attitudes towards problem solving enhances their reflective thinking. Thus, we recommend that teachers should put more effort into increasing their interest and attitude toward problem solving.

Researchers Contribution Ratio

Both authors share the same amount of contribution in the recent article.

Declaration of Conflict of Interest

The authors declare that they have no conflict of interest.

REFERENCES

Arnold, C. L. (1992). An introduction to hierarchical linear models. *Measurement and Evaluation in Counseling and Development*, 25, 58–90.

Aydın, M., & Çelik, T. (2013). Sosyal bilgiler öğretmen adaylarının yansıtıcı düşünme becerilerinin bazı değişkenler açısından incelenmesi. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi,* 34 (2), 169-181.

Barnhart, T. and Es, E. v. (2015). Studying teacher noticing: examining the relationship among pre-service science teachers' ability to attend, analyze and respond to student thinking. *Teaching and Teacher Education*, 45, 83-93. https://doi.org/10.1016/j.tate.2014.09.005

Boyd, E. M., & Fales, A. W. (1983) Reflective learning: Key to learning from experience. *Journal of Humanistic Psychology*, 23, (2,) pp.99-117.

Bryk, A. S., Raudenbush, S. W., & Congdon, R. (2010). HLM7 for Windows [Computer software]. Chicago, IL: Scientific Software International, Inc.

Can, S. (2015). Pre-service science teachers reflective thinking skills toward problem solving. *Educational Research and Reviews*, 10(10), 1449-1457. https://doi.org/10.5897/err2015.2228

Ceylan, G. (2014*). Üniversite öğrencilerinin yansıtıcı düşünme düzeyleri ve araştırmaya yönelik kaygılarının çeşitli değişkenler açısından cart analizi ile incelenmesi* [Master's thesis]. Van Yücüncü Yıl University, Van.

Çanakçı, O., & Özdemir, A. Ş. (2011). Matematik problemi çözme tutum ölçeğinin geliştirilmesi. *Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi, 11(1)*.

Demirel, M., Derman, İ., & Karagedik, E. (2015). A study on the relationship between reflective thinking skills towards problem solving and attitudes towards mathematics. *Procedia- Social and Behavioral Sciences*, 197, 2086-2096. https://doi.org/10.1016/j.sbspro.2015.07.326

Dewey, J. (1933). *How we think*. Prometheus Books.

Duban, N., & Yelken, T. Y. (2010). Öğretmen adaylarının yansıtıcı düşünme eğilimleri ve yansıtıcı öğretmen özellikleriyle ilgili görüşleri. *Çukurova Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *19*(2).

Epstein, A. S. (2003). How planning and reflection develop young children's thinking skills. *Young Children*, *58*(5), 28-36.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). *How to design and evaluate research in education* (8th ed.). McGraw-Hill Humanities.

Gedik, H., Akhan, N. E., & Kılıçoğlu, G. (2014). Sosyal bilgiler öğretmen adaylarının yansıtıcı düşünme eğilimleri. *Mediterranean Journal of Humanities*, 4(2), 113-113. https://doi.org/10.13114/mjh.201428432

Gelman, A., & Hill, J. (2006). *Data analysis using regression and multilevel/hierarchical models*. Cambridge University Press.

Gencel, İ. E., & Saracaloğlu, A. S. (2018). The effect of layered curriculum on reflective thinking and on self-directed learning readiness of prospective teachers. *International Journal of Progressive Education*, 14(1), 8-20. https://doi.org/10.29329/ijpe.2018.129.2

Groom, B., & Maunonen-Eskelinen, I. (2006). The use of portfolios to develop reflective practice in teacher training: a comparative and collaborative approach between two teacher training providers in the UK and Finland. *Teaching in Higher Education*, 11(3), 291-300.

Hassan, S. R., Rosli, R., & Zakaria, E. (2016). The use of i-think map and questioning to promote higher-order thinking skills in mathematics. *Creative Education*, 07(07), 1069-1078. https://doi.org/10.4236/ce.2016.77111

Hatton, N., & Smith, D. D. (1995). Reflection in teacher education: towards definition and implementation. *Teaching and Teacher Education*, 11(1), 33-49. https://doi.org/10.1016/0742-051x(94)00012-u

Hayden, H. E., & Chiu, M. M. (2013). Reflective teaching via a problem exploration–teaching adaptations–resolution cycle. *Journal of Mixed Methods Research*, 9(2), 133-153. https://doi.org/10.1177/1558689813509027

Hegedus, S. J. (2002). *The Nature of reflective thinking in multivariable calculus*. Columbus: ERIC/CSMEE Publications.

Hofmann, D. A. (1997). An Overview of the Logic and Rationale of Hierarchical Linear Models. *Journal of Management*, 23(6), 723–744. https://doi.org/10.1177/014920639702300602

Hox, J. J. (2010). *Multilevel analysis: Techniques and applications* (2nd ed, Quantitative methodology series). Routledge.

Hox, J., & Roberts, J. K. (2011). *Handbook of Advanced Multilevel Analysis*. Routledge. https://doi.org/10.4324/9780203848852

Kember, D. (2008). *Reflective teaching and learning in the health professions: Action research in professional education.* John Wiley & Sons.

Kheirzadeh, S., & Sistani, N. (2018). The effect of reflective teaching on Iranian ELF students' achievement: the case of teaching experience and level of education. *Australian Journal of Teacher Education*, 43(2), 143-156. https://doi.org/10.14221/ajte.2018v43n2.8

Kırnık, D. (2010). *İlköğretim 5. sınıf Türkçe dersinde yansıtıcı düşünmeyi geliştirici etkinliklerin öğrenci başarısına etkisi* [Unpublished master's thesis]. Fırat University: Elazığ.

Kızılkaya, G., & Aşkar, P. (2009). Problem çözmeye yönelik yansıtıcı düşünme becerisi ölçeğinin geliştirilmesi. *Eğitim ve Bilim*, *34*(154), 82-92.

Leeuw, J. d., & Meijer, E. (2008). *Handbook of multilevel analysis*. Springer.

Maas, C. J. M., & Hox, J. J. (2005). Sufficient sample sizes for multilevel modeling. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences*, *1*(3), 86–92. doi:10.1027/1614-2241.1.3.86

Oğuz, A. (2013). Öğrenen özerkliğini destekleme ölçeği'nin geliştirilmesi. *Kuram ve Uygulamada Eğitim Bilimleri*, 13(4).

Orakcı, Ş., Dilekli, Y., & Erdağ, C. (2020). The structural relationship between accountability felt and responsible teaching in turkish teachers: The mediating effect of innovative thinking. *Thinking Skills and Creativity*, 36, 100662. https://doi.org/10.1016/j.tsc.2020.100662

Rakhmawati, Y. and Mustadi, A. (2021). Examining the necessity of reflective module: literacy numeracy skill of students elementary school. *AL-ISHLAH: Jurnal Pendidikan*, 13(1), 597-609. https://doi.org/10.35445/alishlah.v13i1.534

Saygılı, G., & Atahan, R. (2014). Üstün zekâlı çocukların problem çözmeye yönelik yansıtıcı düşünme becerilerinin çeşitli değişkenler bakımından incelenmesi. *Süleyman Demirel Üniversitesi Fen-Edebiyat Fakültesi Sosyal Bilimler Dergisi*, (31).

Töman, U. (2017). Investigation to improve the process of pre-service teachers' reflective thinking skills through an action research. *Universal Journal of Educational Research*, 5(9), 1535-1548. https://doi.org/10.13189/ujer.2017.050911

Ünver, G. (2003). *Yansıtıcı düşünme*. PegemA Yayıncılık.

Woldt, J. L., &Nenad, M. W. (2021). Reflective writing in dental education to improve critical thinking and learning: a systematic review. *Journal of Dental Education*, 85(6), 778-785. https://doi.org/10.1002/jdd.12561

Wopereis, I., Sloep, P., & Poortman, S. (2010). Weblogs as instruments for reflection on action in teacher education. *Interactive Learning Environments*, 18(3), 245-261. https://doi.org/10.1080/10494820.2010.500530

Yeşilbursa, A. (2011). Reflection at the interface of theory and practice: an analysis of pre-service English language teachers' written reflections. *Australian Journal of Teacher Education*, 36(3). https://doi.org/10.14221/ajte.2011v36n3.5

Yıldırım, Ö. (2012). *Okuduğunu anlama başarısıyla ilişkili faktörlerin aşamalı doğrusal modellemeyle belirlenmesi. PISA 2009 Hollanda, Kore ve Türkiye karşılaştırması.* [Doctoral dissertation]. Ankara University, Ankara.